A Robust Two-Step Method for Solving Interval Linear Programming Problems within an Environmental Management Context

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ABSTRACT. In this study, a robust two-step method (RTSM) is developed to solve the interval linear programming (ILP) problem. It improved upon the two-step method (TSM) proposed by Huang et al. (1992) through incorporating additional constraints into solution procedures to avoid absolute violation. RTSM was applied to a simple case related to environmental management. The results demonstrated its applicability of the developed methodology. Compare with the modified interval linear programming (MILP) method proposed by Zhou et al., (2008) and the three-step method (ThSM) developed by Cao and Huang (2011), RTSM can generate a relatively larger solution space and thus avoid significant loss of decision-related information. Besides, RTSM has simpler solution procedures than ThSM, and will not lead to great computational requirement.

Keywords: decision support, algorithm, interval number, optimization, uncertainty

1. Introduction

Optimization is a useful tool for supporting effective environmental management (van Beek et al., 1992; Bloemhof-Ruwaard, et al., 1995; Huang and Chang 2003). However, in many real-world environmental management problems, uncertainties exist in various system components and their interrelationships. For example, waste generation rate within a city is related to many socio-economic and environmental factors, and exhibits uncertain and dynamic features; the efficiency of a municipal wastewater treatment plant is affected by wastewater flow rate, and is uncertain in nature; regional air quality is mainly influenced by air pollutant emissions within this area, which also present uncertain characteristics. Such uncertainties can lead to increased complexities in the related optimization efforts. Simply ignoring these uncertainties is considered undesired as it may result in inferior or wrong decisions (Ruszcyński, 1997; Young, 2001; Ozdemir and Saaty, 2006; Zou, et al., 2010). Therefore, inexact optimization methods are desired for supporting environmental management under uncertainty.

In the past decades, amounts of optimization methods were developed for environmental management under uncertainty, mainly including stochastic, fuzzy and interval mathematical programming (abbreviated as FMP, SMP and IMP, respectively) (Chang and Wang, 1997; Huang et al., 1992; Chang and Lu, 1997; Karmakar and Mujumdar, 2006a, b; Li et al., 2006,

2007, 2008; Nie et al., 2007; Lv et al., 2010; Sun and Huang, 2010; Yan et al., 2010; Fan et al., 2009, 2012a, 2012b). Among them, IMP was an effective approach in dealing with uncertainties, since it does not require distributional information for input parameters (Tong, 1994; Chinneck and Ramada, 2000; Sengupta et al., 2001; Fiedler et al., 2006; Oliveira and Antunes, 2007). Huang et al. (1992) proposed an interval linear programming (ILP) method through incorporating interval numbers into a linear programming (LP) framework and applied it to solid waste management. After that, many inexact LP methods were proposed for dealing with a variety of uncertainties in environmental management problems (Nie et al., 2007; He et al., 2008; Liu et al., 2008).

The two-step method (TSM) proposed by Huang et al. (1992) was mostly used to solve IMP problems. In this method, an ILP problem was converted into two sub-problems that correspond to lower and upper bounds of its objective-function value (Huang et al., 1992). TSM is effective in solving ILP problems without significantly increased computational requirements. However, the main disadvantage of TSM is the potential existence of constraint violation in its solution space. Consequently, Zhou et al. (2008) and Huang and Cao (2011) improved the algorithm of Huang et al. (1992). In detail, Zhou et al. (2008) developed a modified interval linear programming (MILP) method, through incorporating additional constraints into the lower- and upper-bound submodels. Cao and Huang (2011) proposed a three-step method (ThSM) for solving the ILP problem; following a feasibility test, a constricting process for the decision space was implemented, based on the original space from the TSM solutions. The MILP and ThSM could thus help avoid constraint violation in the ILP solution space. However, they were based on relatively harsh constraints that

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led to significantly shrunk decision space, resulting in loss of many feasible decision alternatives. In addition, ThSM might also be associated with issues of infeasibility in the solution process (particularly when the mid-value TSM solutions are infeasible), as well as significantly increased computation requirement. Therefore, a more effective algorithm is desired.

The objective of this study is to develop a robust two-step method (RTSM) for solving ILP problems. The RTSM can help at least guarantee feasibility of the generated solutions under the best-case constraints, while the dimensions of the decision space are not significantly compromised. This is meaningful since, within such a space, all decision alternatives generated through various trade-off analyses will be guaranteed to be potentially feasible points with different feasibility levels. The RTSM also possesses the advantage of convenience for practical applications, since it does not lead to more complicated solution procedures. Finally, a simplified example will be developed to illustrate the solution process.

2. Review of the Existing Solution Methods for ILP

According to Huang (1998), an ILP model can be presented as follows:

$$\text{Max} \quad f^{\pm} = C^{\pm} X^{\pm} \tag{1a}$$

subject to:

$$A^{\pm} X^{\pm} \leq B^{\pm} \tag{1b}$$

$$X^{\pm} \geq 0 \tag{1c}$$

where $A^{\pm} \in \{R^{\pm}\}^{m \times n}$, $C^{\pm} \in \{R^{\pm}\}^{1 \times n}$, $B^{\pm} \in \{R^{\pm}\}^{m \times 1}$, $X^{\pm} \in \{R^{\pm}\}^{n \times 1}$; $R^{\pm}$ denotes a set of interval numbers; $A^{\pm} = (a_{ij}^{\pm})_{m \times n}$, $C^{\pm} = (c_1^{\pm}, c_2^{\pm}, ..., c_n^{\pm})$, $B^{\pm} = (b_1^{\pm}, b_2^{\pm}, ..., b_m^{\pm})^T$ and $X^{\pm} = (x_1^{\pm}, x_2^{\pm}, ..., x_n^{\pm})^T$. An interval number ($a^{\pm}$) is defined as (Huang et al., 1992): $a^{\pm} = [a^-, a^+] = \{t \in a \mid a^- \leq t \leq a^+\}$.

To solve model (1), Huang et al. (1992) proposed a two-step method (TSM) through analyzing interrelationships among parameters and variables in the objective and constraints. The main idea of TSM is to convert the original ILP model into two LP submodels which correspond to the lower and upper bounds of the objective-function value. In detail, for n interval coefficients $c_j^{\pm}$ ($j = 1, 2, ..., n$) in the objective function, the former k coefficients are assumed to be positive (i.e. $c_j^{\pm} \geq 0$, for $j = 1, 2, ..., k$), and the latter (n - k) coefficients are negative (i.e. $c_j^{\pm} \leq 0$, for $j = k + 1, k + 2, ..., n$). Thus the first submodel of model (1) would correspond to f^+. It can be formulated as follows (assume that $b_i^{\pm} > 0$ and $f^{\pm} > 0$):

$$\text{Max} \quad f^+ = \sum_{j=1}^{k} c_j^+ x_j^+ + \sum_{j=k+1}^{n} c_j^+ x_j^- \tag{2a}$$

subject to:

$$\sum_{j=1}^{k} \left| a_{ij}^{\pm} \right|^- Sign(a_{ij}^{\pm}) x_j^+ + \sum_{j=k+1}^{n} \left| a_{ij}^{\pm} \right|^+ Sign(a_{ij}^{\pm}) x_j^- \leq b_i^+, \quad i = 1, 2, ..., m. \tag{2b}$$

$$x_j^+ \geq 0, \quad j = 1, 2, ..., k \tag{2c}$$

$$x_j^- \geq 0, \quad j = k+1, k+2, ..., n \tag{2d}$$

Solutions of x_{jopt}^+ ($j = 1, 2, ..., k$) and x_{jopt}^- ($j = k + 1, k + 2, ..., n$) can be obtained through solving submodel (2). Based on the solutions of model (2), the submodel corresponding to f^- can be formulated as follows (assume that $b_i^{\pm} > 0$ and $f^{\pm} > 0$):

$$\text{Max} \quad f^- = \sum_{j=1}^{k} c_j^- x_j^- + \sum_{j=k+1}^{n} c_j^- x_j^+, \tag{3a}$$

subject to:

$$\sum_{j=1}^{k} \left| a_{ij}^{\pm} \right|^+ Sign(a_{ij}^{\pm}) x_j^- + \sum_{j=k+1}^{n} \left| a_{ij}^{\pm} \right|^- Sign(a_{ij}^{\pm}) x_j^+ \leq b_i^-, \quad i = 1, 2, ..., m. \tag{3b}$$

$$0 \leq x_j^- \leq x_{j\,opt}^+, \quad j = 1, 2, ..., k. \tag{3c}$$

$$x_j^+ \geq x_{j\,opt}^-, \quad j = k+1, k+2, ..., n. \tag{3d}$$

From model (3), solutions of x_{jopt}^- ($j = 1, 2, ..., k$) and x_{jopt}^+ ($j = k + 1, k + 2, ..., n$) can be obtained. Thus, the final solutions of $f_{opt}^{\pm} = [f_{opt}^-, f_{opt}^+]$ and $x_{jopt}^{\pm} = [x_{jopt}^-, x_{jopt}^+]$ can be obtained for model (1).

The TSM has been widely used for solving many inexact environmental management problems that can be formulated as ILP models (Huang et al., 1995). However, its main weakness is the potential violation of the best-case constraints when the detailed decision point varies within the generated decision space. For example, consider an ILP model: max $\{c_1^{\pm} x_1^{\pm} - c_2^{\pm} x_2^{\pm} \mid a_{11}^{\pm} x_1^{\pm} + a_{12}^{\pm} x_2^{\pm} \leq b_1^{\pm}, a_{21}^{\pm} x_1^{\pm} + a_{22}^{\pm} x_2^{\pm} \leq b_2^{\pm}\}$, and assume that all of its coefficients are non-negative. According to TSM, constraints $a_{i1}^+ x_1^+ + a_{i2}^+ x_2^+ \leq b_i^-$ and $a_{i1}^- x_1^- + a_{i2}^- x_2^- \leq b_i^+$ correspond to the lower and upper bounds of the objective-function value, respectively. However, these two constraints cannot fully guarantee feasibility of the best-case constraints (i.e. $a_{i1}^- x_1 + a_{i2}^- x_2 \leq b_i^+$) for any $x_1 \in [x_1^-, x_1^+]$ and $x_2 \in [x_2^-, x_2^+]$.

Zhou et al. (2008) modified the TSM algorithm through introducing additional constraints into the lower- and upper-bound submodels. For model (1), their modified ILP (or MILP) solution procedures are presented as follows:

Submodel 1

$$\text{Max} \quad f^+ = \sum_{j=1}^{k} c_j^+ x_j^+ + \sum_{j=k+1}^{n} c_j^+ x_j^- \tag{4a}$$

subject to:

$$\sum_{j=1}^{k} \left|a_{ij}^{\pm}\right|^{-} Sign(a_{ij}^{\pm})x_j^+ + \sum_{j=k+1}^{n} \left|a_{ij}^{\pm}\right|^+ Sign(a_{ij}^{\pm})x_j^- \le b_i^+ , i = 1, 2, \ldots, m. \tag{4b}$$

$$x_j^+ \ge 0 , j = 1, 2, \ldots, k \tag{4c}$$

$$x_j^- \ge 0 , j = k+1, k+2, \ldots, n \tag{4d}$$

Submodel 2

$$\text{Max } f^- = \sum_{j=1}^{k} c_j^- x_j^- + \sum_{j=k+1}^{n} c_j^- x_j^+ , \tag{5a}$$

subject to:

$$\sum_{j=1}^{k} \left|a_{ij}^{\pm}\right|^+ Sign(a_{ij}^{\pm})x_j^- + \sum_{j=k+1}^{n} \left|a_{ij}^{\pm}\right|^- Sign(a_{ij}^{\pm})x_j^+ \le b_i^- , i = 1, 2, \ldots, m. \tag{5b}$$

$$\sum_{j=k-p+1}^{k} -(\left|a_{\delta j}^{\pm}\right|^+ x_j^- - \left|a_{\delta j}^{\pm}\right|^- x_{jopt}^+) + \sum_{j=n-q+1}^{n} (\left|a_{\delta j}^{\pm}\right|^- x_j^+ - \left|a_{\delta j}^{\pm}\right|^+ x_{jopt}^-) \le 0 \tag{5c}$$

$$0 \le x_j^- \le x_{j\,opt}^+ , j = 1, 2, \ldots, k . \tag{5d}$$

$$x_j^+ \ge x_{j\,opt}^- , j = k+1, k+2, \ldots, n . \tag{5e}$$

where δ is the number of constraints in Equation (4b) that meet $\sum_{j=1}^{k} |a_{\delta j}^{\pm}| Sign(a_{\delta j}^{\pm})x_{jopt}^+ + \sum_{j=k+1}^{n} |a_{\delta j}^{\pm}|^+ Sign(a_{\delta j}^{\pm})x_{jopt}^- = b_{\delta}^+$ and $a_{\delta j}^{\pm} \le 0$ for $j = k-p+1, k-p+2, \ldots, k$ and $a_{\delta j}^{\pm} \ge 0$ for $j = n -q+1, n-q+2, \ldots, n$.

The MILP method can ensure the ILP solution space to be absolutely feasible, without violation of the best-case constraints. However, the MILP was based on relatively harsh constraints (i.e. additional constraints to submodel 5) that led to significantly shrunk decision space, resulting in loss of many feasible decision alternatives; moreover, uncertainty of the objective function may be enlarged due to the additional constraints in submodel (5).

Cao and Huang (2011) proposed a three-step method (ThSM) for solving the ILP problem; following a feasibility test, a constricting process for the decision space was implemented, based on the original space from the TSM solutions. The ThSM consisted of the following steps (Cao and Huang, 2011):

Step 1: generate solutions of model (1) through TSM.

Step 2: conduct feasibility test to analyze whether the TSM solutions can satisfy the best-case constraints. For constraint i ($i = 1, 2, \ldots, m$), the idea of feasibility test is to identify $X^* \in X_{opt}^{\pm}$ such that $\sum_{j=1}^{n} a_{ij}^- x_j^* = \max\{\sum_{j=1}^{n} a_{ij}^- x_j \mid x_j \in x_j^{\pm}\}$; if

$\sum_{j=1}^{n} a_{ij}^- x_j^* \le b_i^+$ is tenable, then $X_{opt}^{\pm}$ pass the feasibility test; otherwise, it means that part of solutions from $X_{opt}^{\pm}$ will violate the constraints.

Step 3: eliminate the infeasible solution zone through a constricting approach. For solutions that do not pass the feasibility test, two constricting approaches are proposed to remove them:

ThSM-I

Max q (6a)

subject to:

$$\sum_{j=1}^{p_i} a_{ij}^- q d_j - \sum_{j=p_i+1}^{n_0} a_{ij}^- q d_j \le b_i^+ - \sum_{j=1}^{n} a_{ij}^- m_j , i = 1, 2, \ldots, m. \tag{6b}$$

$$0 \le q \le 1 \tag{6c}$$

ThSM-II

Max $q_1 \times q_2 \times \ldots \times q_{n_0}$ (7a)

subject to:

$$\sum_{j=1}^{p_i} a_{ij}^- q_j d_j - \sum_{j=p_i+1}^{n_0} a_{ij}^- q_j d_j \le b_i^+ - \sum_{j=1}^{n} a_{ij}^- m_j , i = 1, 2, \ldots, m. \tag{7b}$$

$$0 \le q_j \le 1 \tag{7c}$$

where $m_j = 0.5(x_{jopt}^- + x_{jopt}^+)$; $d_j = 0.5(x_{jopt}^+ - x_{jopt}^-)$; $j = 1, 2, \ldots, n_0$; n_0 means the number of interval solutions obtained through submodels (2) and (3).

The main advantage of the ThSM method is its ability to avoid constraint violation in the ILP solution space. However, many feasible solutions will be eliminated in the constricting process which compresses the solution space from all dimensions. Particularly, it leads to loss of the solution zones that are close to the worst-case constraints. In addition, ThSM might also be associated with issues of infeasibility in the solution process. For example, when the mid-value of TSM solutions [i.e. the solution set defined by $x_j^m = 0.5(x_{jopt}^- + x_{jopt}^+), \forall j$] does not pass feasibility test, then models (6) and (7) will become infeasible. Also, ThSM may lead to significantly increased computation requirement, which may hinder its practical applications, especially for problems with a large number of constraints.

3. Robust Two-Step Method

A robust two-step method (RTSM) is proposed for solving ILP problems, where the best-cases constraints of model (1) will be incorporated within the submodel corresponding to f^+ (when the objective is to be maximized). This can thus help

avoid absolute constraint violation (i.e. violations even beyond the best-case constraints) as the decision variables fluctuated within the generated decision space.

Lemma 1 (Huang et al. 1995): For $A \in [A^-, A^+]$ and $B \in [B^-, B^+]$, denoting $Q = \{X \mid AX \le B, X \ge 0\}$, $Q^- = \{X \mid A^+X \le B^-, X \ge 0\}$, and $Q^+ = \{X \mid A^-X \le B^+, X \ge 0\}$, we have $Q^+ \supseteq Q \supseteq Q^-$:

Proof: If both $X \in Q^-$ and $X \ge 0$ hold, we have $AX \le A^+X \le B^- \le B$, such that $X \in Q$ holds. Furthermore, if both $X \in Q$ and $X \ge 0$ hold, we have $A^-X \le AX \le B \le B^+$, such that $X \in Q^+$ holds. Hence, $Q^+ \supseteq Q \supseteq Q^-$.

Remark 1: Lemma 1 provides the relationships between feasible X sets under two extreme constraints (best and worst cases).

Lemma 2 (Huang et al., 1995): For n interval coefficients $c_j^\pm$ ($j = 1, 2, \ldots, n$) in the objective function of model (1), if k_1 of them are positive, and k_2 are negative, let the former k_1 coefficients be positive, i.e. $c_j^\pm \ge 0$ ($j = 1, 2, \ldots, k_1$) and the latter k_2 coefficients be negative, i.e. $c_j^\pm \le 0$ ($j = k_1 + 1, k_1 + 2, \ldots, n$), where $k_1 + k_2 = n$ (the model does not include the situation in which the two bounds of $c_j^\pm$ have different signs). Thus, the following expression can be developed for the lower and upper bounds of $f^\pm$:

$$f^+ = \sum_{j=1}^{k_1} c_j^+ x_j^+ + \sum_{j=k_1+1}^{n} c_j^+ x_j^- \tag{8a}$$

$$f^- = \sum_{j=1}^{k_1} c_j^- x_j^- + \sum_{j=k_1+1}^{n} c_j^- x_j^+ \tag{8b}$$

Proof: Straightforward.

Corollary 1: the largest feasible area corresponding to f^+ can be formed as follows:

$$Q_{max}^+ = \{x_j^+ \ge 0, j = 1, 2, \ldots, k_1; x_j^- \ge 0, j = k_1+1, k_1+2, \ldots, n; \mid \sum_{j=1}^{k_1}$$

$$a_{ij}^- x_j^+ + \sum_{j=k_1+1}^{n} a_{ij}^- x_j^- \le b_i^+, \forall i\} \tag{9a}$$

Proof: Assume $a_{ij}^u \in [a_{ij}^-, a_{ij}^+]$ and $Q_u^+ = \{x_j^+ \ge 0, j = 1, 2, \ldots, k_1; x_j^- \ge 0, j = k_1 + 1, k_1 + 2, \ldots, n \mid \sum_{j=1}^{k_1} a_{ij}^u x_j^+ + \sum_{j=k_1+1}^{n} a_{ij}^u x_j^- \le b_i^+, \forall i\}$. For any $X_0 = (x_{j0}^+ \ge 0, j = 1, 2, \ldots, k_1; x_{j0}^- \ge 0, j = k_1 + 1, k_1 + 2, \ldots, n) \in Q_u^+$, $\sum_{j=1}^{k_1} a_{ij}^u x_{j0}^+ + \sum_{j=k_1+1}^{n} a_{ij}^u x_{j0}^- \le b_i^+$ holds. Because $a_{ij}^- \le a_{ij}^u \le a_{ij}^+$ and $x_{j0} \ge 0$, we have $\sum_{j=1}^{k_1} a_{ij}^- x_{j0}^+ + \sum_{j=k_1+1}^{n} a_{ij}^- x_{j0}^- \le \sum_{j=1}^{k_1} a_{ij}^u x_{j0}^+ + \sum_{j=k_1+1}^{n} a_{ij}^u x_{j0}^- \le b_i^+, \forall i$. Therefore, $X_0 \in Q_{max}^+$, namely $Q_u^+ \subseteq Q_{max}^+$.

Corollary 2: the smallest feasible area corresponding f^- is:

$$Q_{min}^- = \{x_j^- \ge 0, j = 1, 2, \ldots, k_1; x_j^+ \ge 0, j = k_1+1, k_1+2, \ldots, n; \mid \sum_{j=1}^{k_1}$$

$$a_{ij}^+ x_j^- + \sum_{j=k_1+1}^{n} a_{ij}^+ x_j^+ \le b_i^-\} \tag{9b}$$

Proof: Assume $a_{ij}^l \in [a_{ij}^-, a_{ij}^+]$ and $Q^- = \{x_j^- \ge 0, j = 1, 2, \ldots, k_1; x_j^+ \ge 0, j = k_1 + 1, k_1 + 2, \ldots, n \mid \sum_{j=1}^{k_1} a_{ij}^l x_j^- + \sum_{j=k_1+1}^{n} a_{ij}^l x_j^+ \le b_i^-, \forall i\}$. Consider $\forall X = (x_j^- \ge 0, j = 1, 2, \ldots, k_1; x_j^+ \ge 0, j = k_1 + 1, k_1 + 2, \ldots, n) \in Q_{min}^-$, then $\sum_{j=1}^{k_1} a_{ij}^+ x_j^- + \sum_{j=k_1+1}^{n} a_{ij}^+ x_j^+ \le b_i^+$ holds. Since $a_{ij}^- \le a_{ij}^l \le a_{ij}^+$ and $x_j^\pm \ge 0$, we have $\sum_{j=1}^{k_1} a_{ij}^l x_j^- + \sum_{j=k_1+1}^{n} a_{ij}^l x_j^+ \le \sum_{j=1}^{k_1} a_{ij}^+ x_j^- + \sum_{j=k_1+1}^{n} a_{ij}^+ x_j^+ \le b_i^-, \forall i$. Therefore, $X \in Q_l^-$, namely $Q_{min}^- \subseteq Q_l^-$.

Theorem 1: (Huang et al., 1995) In order to obtain interval solutions expressed as $f_{opt}^\pm = [f_{opt}^-, f_{opt}^+]$ and $x_{jopt}^\pm = [x_{jopt}^-, x_{jopt}^+]$, constraints corresponding to f^+ can be developed as follows:

$$\sum_{j=1}^{k} \left|a_{ij}^\pm\right|^- Sign(a_{ij}^\pm) x_j^+ + \sum_{j=k+1}^{n} \left|a_{ij}^\pm\right|^+ Sign(a_{ij}^\pm) x_j^- \le b_i^+, i = 1, 2, \ldots, m \tag{10a}$$

Similarly, the relevant constraints corresponding to f^- can also be developed as follows:

$$\sum_{j=1}^{k} \left|a_{ij}^\pm\right|^+ Sign(a_{ij}^\pm) x_j^- + \sum_{j=k+1}^{n} \left|a_{ij}^\pm\right|^- Sign(a_{ij}^\pm) x_j^+ \le b_i^-, i = 1, 2, \ldots, m \tag{10b}$$

Lemma 3: Consider constraint i in equation (1b), $\sum_{j=1}^{n} a_{ij}^\pm x_j^\pm \le b_i^\pm$, where $a_{ij}^\pm \ge 0$ ($j = 1, 2, \ldots, l_{i1}$) and $a_{ij}^\pm \le 0$ ($j = l_{i1} + 1, l_{i1} + 2, \ldots, n$). For any $x_j \in [x_j^-, x_j^+]$ ($j = 1, 2, \ldots, n$), the best-case constraint of constraint i (i.e. $\sum_{j=1}^{n} a_{ij} x_j \le b_i^+$) can be satisfied, if and only if:

$$\sum_{j=1}^{l_{i1}} a_{ij}^- x_j^+ + \sum_{j=l_{i1}+1}^{n} a_{ij}^- x_j^- \le b_i^+ \tag{11}$$

Proof: Since $a_{ij}^\pm \ge 0$ ($j = 1, 2, \ldots, l_{i1}$) and $a_{ij}^\pm \le 0$ ($j = l_{i1} + 1, l_{i1} + 2 \ldots, n$), then for any $x_j \in [x_j^-, x_j^+] \ge 0$ ($j = 1, 2, \ldots, n$), $x_j \le x_j^+$ and $x_j \ge x_j^-$ hold. Thus we have $a_{ij}^- x_j \le a_{ij}^- x_j^+$ ($j = 1, 2, \ldots, l_{i1}$) and $a_{ij}^- x_j \le a_{ij}^- x_j^-$ ($j = l_{i1} + 1, l_{i1} + 2 \ldots, n$). If $\sum_{j=1}^{l_{i1}} a_{ij}^- x_j^+ + \sum_{j=l_{i1}+1}^{n} a_{ij}^- x_j^- \le b_i^+$, then we have $\sum_{j=1}^{l_{i1}} a_{ij}^- x_j + \sum_{j=l_{i1}+1}^{n} a_{ij}^- x_j \le b_i^+$.

Theorem 2: Consider constraint i_0 of model (1):

$$\sum_{j=1}^{k_1} a_{i_0 j}^\pm x_j^+ + \sum_{j=k_1+1}^{n} a_{i_0 j}^\pm x_j^\pm \le b_{i_0}^\pm \tag{12}$$

Three criteria exist for determining whether additional constraints are needed to avoid violation of the best-case constraint:

(1) If $a_{i_0 j}^\pm \times c_j^\pm \ge 0 \; \forall j$, then solution $x_{jopt}^\pm = [x_{jopt}^-, x_{jopt}^+]$ obtained from submodels (2) and (3) can ensure that for any $x_j \in [x_{jopt}^-, x_{jopt}^+]$ ($j = 1, 2, \ldots, n$), $\sum_{j=1}^{k_1} a_{i_0 j}^- x_j + \sum_{j=k_1+1}^{n} a_{i_0 j}^- x_j \le b_{i_0}^+$ is satisfied.

(2) If $a_{i_0 j}^{\pm} \times c_j^{\pm} \leq 0 \ \forall j$, then the solutions $x_{jopt}^{\pm} = [x_{jopt}^-, x_{jopt}^+]$ obtained through submodels (2) and (3) can guarantee that, for any $x_j \in [x_{jopt}^-, x_{jopt}^+]$ ($j = 1, 2, \ldots, n$), $\sum_{j=1}^{k_1} a_{i_0 j}^+ x_j + \sum_{j=k_1+1}^{n} a_{i_0 j}^- x_j \leq b_{i_0}^+$ is satisfied.

(3) If $a_{i_0 j}^{\pm} \times c_j^{\pm} \geq 0$ [$j = 1, 2, \ldots, l_{i_0 1}$ ($l_{i_0 1} \leq k$) and $k+1, k+2 \ldots, l_{i_0 2}$ ($l_{i_0 2} > k$)] and $a_{i_0 j}^{\pm} \times c_j^{\pm} \leq 0$ [$j = l_{i_0 1} + 1, l_{i_0 1} + 2, \ldots, k$ and $l_{i_0 2} + 1, l_{i_0 2} + 2, \ldots, n$], then solutions $x_{jopt}^{\pm} = [x_{jopt}^-, x_{jopt}^+]$ obtained from submodels (2) and (3) will not lead to violation of the best-case constraint ($\sum_{j=1}^{n} a_{ij}^- x_j \leq b_i^-$), if and only if:

$$\sum_{j=1}^{l_{i_0 1}} a_{i_0 j}^- x_j^+ + \sum_{j=l_{i_0 1}+1}^{k} a_{i_0 j}^- x_{jopt}^- + \sum_{j=k+1}^{l_{i_0 2}} a_{i_0 j}^- x_j^- + \sum_{j=l_{i_0 2}+1}^{n} a_{i_0 j}^- x_{jopt}^+ \leq b_{i_0}^+ \tag{13}$$

where x_{jopt}^- and x_{jopt}^+ are solutions obtained from submodel (3) which corresponds to f^-.

Proof:

(1) From model (2), we have: $\sum_{j=1}^{k} |a_{i_0 j}^{\pm}|^- Sign(a_{i_0 j}^{\pm}) x_j^+ + \sum_{j=k+1}^{n} |a_{i_0 j}^{\pm}|^+ Sign(a_{i_0 j}^{\pm}) x_j^- \leq b_{i_0}^+$. Since $a_{i_0 j}^{\pm} \times c_j^{\pm} \geq 0 \ \forall j$, we have: $a_{i_0 j}^{\pm} \geq 0$ ($j = 1, 2, \ldots, k$) and $a_{i_0 j}^{\pm} \leq 0$ ($j = k+1, k+2, \ldots, n$). Thus, $|a_{i_0 j}^{\pm}|^- Sign(a_{i_0 j}^{\pm}) x_j^+ = a_{i_0 j}^- x_j^+$, and $|a_{i_0 j}^{\pm}|^+ Sign(a_{i_0 j}^{\pm}) x_j^- = a_{i_0 j}^- x_j^-$ hold. Therefore, we have $\sum_{j=1}^{k} |a_{i_0 j}^{\pm}|^- Sign(a_{i_0 j}^{\pm}) x_j^+ + \sum_{j=k+1}^{n} |a_{i_0 j}^{\pm}|^+ Sign(a_{i_0 j}^{\pm}) x_j^- = \sum_{j=1}^{k} a_{i_0 j}^- x_j^+ + \sum_{j=k+1}^{n} a_{i_0 j}^- x_j^- \leq b_{i_0}^+$. For any $x_j \in [x_j^-, x_j^+]$ ($j = 1, 2, \ldots, n$), $a_{i_0 j}^- x_j \leq a_{i_0 j}^- x_j^+$ ($j = 1, 2, \ldots, k$) and $a_{i_0 j}^- x_j \leq a_{i_0 j}^- x_j^-$ ($j = k+1, k+2, \ldots, n$) hold. Consequently, we have $\sum_{j=1}^{k} a_{i_0 j}^- x_j + \sum_{j=k+1}^{n} a_{i_0 j}^- x_j \leq \sum_{j=1}^{k} a_{i_0 j}^- x_j^+ + \sum_{j=k+1}^{n} a_{i_0 j}^- x_j^- \leq b_{i_0}^+$.

(2) Based on model (3), $\sum_{j=1}^{k} |a_{i_0 j}^{\pm}|^+ Sign(a_{i_0 j}^{\pm}) x_j^- + \sum_{j=k+1}^{n} |a_{i_0 j}^{\pm}|^- Sign(a_{i_0 j}^{\pm}) x_j^+ \leq b_{i_0}^-$ holds. Since $a_{i_0 j}^{\pm} \times c_j^{\pm} \leq 0 \ \forall j$, then we have $a_{i_0 j}^{\pm} \leq 0$ ($j = 1, 2, \ldots, k$) and $a_{i_0 j}^{\pm} \geq 0$ ($j = k+1, k+2 \ldots, n$). Thus, $|a_{i_0 j}^{\pm}|^+ Sign(a_{i_0 j}^{\pm}) x_j^- = a_{i_0 j}^- x_j^-$ and $|a_{i_0 j}^{\pm}|^- Sign(a_{i_0 j}^{\pm}) x_j^+ = a_{i_0 j}^- x_j^+$ hold. Consequently, we have $\sum_{j=1}^{k} |a_{i_0 j}^{\pm}|^+ Sign(a_{i_0 j}^{\pm}) x_j^- + \sum_{j=k+1}^{n} |a_{i_0 j}^{\pm}|^- Sign(a_{i_0 j}^{\pm}) x_j^+ = \sum_{j=1}^{k} a_{i_0 j}^- x_j^- + \sum_{j=k+1}^{n} a_{i_0 j}^- x_j^+ \leq b_{i_0}^- \leq b_{i_0}^+$. For any $x_j \in [x_j^-, x_j^+]$ ($j = 1, 2, \ldots, n$), $a_{i_0 j}^- x_j \leq a_{i_0 j}^- x_j^-$ ($j = 1, 2, \ldots, k$) and $a_{i_0 j}^- x_j \leq a_{i_0 j}^- x_j^+$ ($j = k+1, k+2, \ldots, n$) hold. Therefore, we have $\sum_{j=1}^{k} a_{i_0 j}^- x_j + \sum_{j=k+1}^{n} a_{i_0 j}^- x_j \leq \sum_{j=1}^{k} a_{i_0 j}^- x_j^- + \sum_{j=k+1}^{n} a_{i_0 j}^- x_j^+ \leq b_{i_0}^- \leq b_{i_0}^+$.

(3) If $a_{i_0 j}^{\pm} \times c_j^{\pm} \geq 0$ [$j = 1, 2, \ldots, l_{i_0 1}$ ($l_{i_0 1} \leq k$) and $k+1, k+2, \ldots, l_{i_0 2}$ ($l_{i_0 2} > k$)] and $a_{i_0 j}^{\pm} \times c_j^{\pm} \leq 0$ [$j = l_{i_0 1} + 1, l_{i_0 1} + 2, \ldots, k$ and $l_{i_0 2} + 1, l_{i_0 2} + 2, \ldots, n$], we have $a_{i_0 j}^{\pm} \geq 0$ ($j = 1, 2, \ldots, l_{i_0 1}$ and $l_{i_0 2} + 1, l_{i_0 2} + 2, \ldots, n$) and $a_{i_0 j}^{\pm} \leq 0$ ($j = l_{i_0 1} + 1, l_{i_0 2} + 2, \ldots, k, k+1, \ldots, l_{i_0 2}$). When equation (13) holds, for any $x_j \in [x_j^-, x_j^+]$ ($j = 1, 2, \ldots, n$) and $x_j^- \leq x_j \leq x_j^+$, we have $\sum_{j=1}^{l_{i_0 1}} a_{i_0 j}^- x_j^+ \geq \sum_{j=1}^{l_{i_0 1}} a_{i_0 j}^- x_j$ and $\sum_{j=l_{i_0 2}+1}^{n} a_{i_0 j}^- x_{jopt}^+ \geq \sum_{j=l_{i_0 2}+1}^{n} a_{i_0 j}^- x_j$ due to $a_{i_0 j}^{\pm} \geq 0$ ($j = 1, 2, \ldots, l_{i_0 1}$ and $l_{i_0 2} + 1, l_{i_0 2} + 2, \ldots, n$); meanwhile $\sum_{j=l_{i_0 1}+1}^{k} a_{i_0 j}^- x_{jopt}^- \geq \sum_{j=l_{i_0 1}+1}^{k} a_{i_0 j}^- x_j$ and $\sum_{j=k+1}^{l_{i_0 2}} a_{i_0 j}^- x_j^- \geq \sum_{j=k+1}^{l_{i_0 2}} a_{i_0 j}^- x_j$ will hold since $a_{i_0 j}^{\pm} \leq 0$ ($j = l_{i_0 1} + 1, l_{i_0 1} + 2, \ldots, k, k+1, \ldots, l_{i_0 2}$). Therefore, we have $\sum_{j=1}^{n} a_{i_0 j}^- x_j \leq \sum_{j=1}^{l_{i_0 1}} a_{i_0 j}^- x_j^+ + \sum_{j=l_{i_0 1}+1}^{k} a_{i_0 j}^- x_{jopt}^- + \sum_{j=k+1}^{l_{i_0 2}} a_{i_0 j}^- x_j^- + \sum_{j=l_{i_0 2}+1}^{n} a_{i_0 j}^- x_{jopt}^+ \leq b_{i_0}^+$.

Remark 3: To solve model (1), the submodel corresponding to f^- would be solved firstly because of the following reasons:

(1) From Lemma 1, the solutions of the submodel corresponding to f^- [i.e. x_{jopt}^- ($j = 1, 2, \ldots, k$) and x_{jopt}^+ ($j = k+1, k+2, \ldots, n$)] are identified from set Q^-. Since $Q^- \subseteq Q^+$, $\exists x_j^{\pm} \in Q^+$, such that $x_j^+ \geq x_{jopt}^-$ ($j = 1, 2, \ldots, k$) and $x_j^- \leq x_{jopt}^+$ ($j = k+1, k+2, \ldots, n$).

(2) For model (1), the submodel corresponding to f^- would use relatively strict constraints (i.e. lower-bound values for the constraints' right-hand sides), and the submodel corresponding to f^+ would use relatively relaxed constraints (i.e. upper-bound values for the right-hand sides). From theorem 2, additional constraints based on the solutions corresponding to f^- should be interposed for the submodel corresponding to f^+. This will help avoid violation of the best-case constraints as the decision variables fluctuated within the generated decision space.

Based on the above analysis, a robust two-step method (RTSM) will be developed to solve model (1). In this method, a conservative submodel (i.e. the submodel corresponding to f^- when the objective is to be maximized) will be solved firstly; then an optimistic submodel (i.e. the submodel corresponding to f^+) will be derived based on solutions from the conservative submodel. In detail, the conservative submodel can be presented as follows (assume $b_i^{\pm} > 0$, $f^{\pm} > 0$):

$$\text{Max } f^- = \sum_{j=1}^{k} c_j^- x_j^- + \sum_{j=k+1}^{n} c_j^- x_j^+, \tag{14a}$$

subject to:

$$\sum_{j=1}^{k} |a_{ij}^{\pm}|^+ Sign(a_{ij}^{\pm}) x_j^- + \sum_{j=k+1}^{n} |a_{ij}^{\pm}|^- Sign(a_{ij}^{\pm}) x_j^+ \leq b_i^-, \ i = 1, 2, \ldots, m \tag{14b}$$

$$x_j^- \geq 0, \ j = 1, 2, \ldots, k \tag{14c}$$

$$x_j^+ \geq 0, \ j = k+1, k+2, \ldots, n \tag{14d}$$

From submodel (14), solutions of x_{jopt}^- ($j = 1, 2, \ldots, k$) and x_{jopt}^+ ($j = k+1, k+2, \ldots, n$) can be obtained. These solutions will be incorporated into the optimistic submodel. The third criterion (as shown in Theorem 2) will then be employed to derive additional constraints that can help prevent from violation of the best-case constraints. Thus, the optimistic submodel can be formulated as follows (assume $b_i^{\pm} > 0$, $f^{\pm} > 0$):

$$\text{Max } f^+ = \sum_{j=1}^{k} c_j^+ x_j^+ + \sum_{j=k+1}^{n} c_j^+ x_j^- \tag{15a}$$

subject to:

$$\sum_{j=1}^{k} |a_{ij}^{\pm}|^- Sign(a_{ij}^{\pm}) x_j^+ + \sum_{j=k+1}^{n} |a_{ij}^{\pm}|^+ Sign(a_{ij}^{\pm}) x_j^- \leq b_i^+, \ i = 1, 2, \ldots, m. \tag{15b}$$

$$\sum_{j=1}^{l_{i1}} a_{ij}^- x_j^+ + \sum_{j=l_{i1}+1}^{k} a_{ij}^- x_{jopt}^- + \sum_{j=k+1}^{l_{i2}} a_{ij}^- x_j^- + \sum_{j=l_{i2}+1}^{n} a_{ij}^- x_{jopt}^+ \leq b_i^+ \tag{15c}$$

$$x_j^+ \geq x_{jopt}^- \quad j = 1, 2,..., k \tag{15d}$$

$$x_j^- \leq x_{jopt}^+ \quad j = k+1, k+2,..., n \tag{15e}$$

$$x_j^+ \geq 0, j = 1, 2, ..., k \tag{15c}$$

$$x_j^- \geq 0, j = k+1, k+2, ..., n \tag{15d}$$

where $c_j^\pm \geq 0$ ($j = 1, 2, ..., k$), and $c_j^\pm \leq 0$ ($j = k+1, k+2, ..., n$); $a_{ij}^\pm \geq 0$ ($j = 1, 2, ..., l_{i1}; j = l_{i2}+1, l_{i2}+2,..., n$) and $a_{ij}^\pm \leq 0$ ($j = l_{i1}+1, l_{i1}+2, ..., l_{i2}$), where $l_{i1} \leq k$, and $l_{i2} \geq k$.

Hence, solutions of x_{jopt}^+ ($j = 1, 2, ..., k$) and x_{jopt}^- ($j = k+1, k+2, ..., n$) can be generated through submodel (15). Therefore, the ultimate solutions for model (1) are: $x_{jopt}^\pm = [x_{jopt}^-, x_{jopt}^+]$ and $f_{opt}^\pm = [f_{opt}^-, f_{opt}^+]$.

4. Illustrative Case Study of Environmental Management

An illustrative case of interval parameter linear programming presented by Zhou et al. (2008) would be analyzed to show the solution process of RTSM:

$$\text{Max} \quad f^\pm = [26, 30]x_1^\pm - [5.5, 6.0]x_2^\pm \tag{16a}$$

subject to:

$$[8, 10]x_1^\pm - [12, 14]x_2^\pm \leq [3.8, 4.2] \tag{16b}$$

$$[1.0, 1.1]x_1^\pm + [0.19, 0.2]x_2^\pm \leq [6.5, 7] \tag{16c}$$

$$x_1^\pm \geq 0, \quad x_2^\pm \geq 0 \tag{16d}$$

In the above problem, it is assumed that the objective is to maximize economic return, while constraints (16b) and (16c) denote respectively limitations in resource availability and pollutant emission allowance.

Figure 1 shows a graphical presentation of the decision space for model (16). Based on constraints (16b) and (16c), the decision space is firstly divided into eight zones (i.e. Zones A, B, C, D, E, F, G, H, and I). Different zones would satisfy or violate different constraints. For example, any point in zone C is infeasible for constraint (16b); in contrast, all solutions in zone G satisfy constraint (16b) regardless the variation of its uncertain coefficients. In detail, zones B, C, D, E, and F are infeasible for model (16) since they would lead to absolute infeasibility. This means that the points located in those zones would violate at least one of the two best-case constraints for model (16), which are presented as the dash lines in Figure 1. Conversely, the solutions in zone H are absolutely feasible for model (16) since they satisfy the worst-case constraints; however, these solutions would lead to serious waste of resources since uncertainties of the right-hand-side coefficients are not consi-
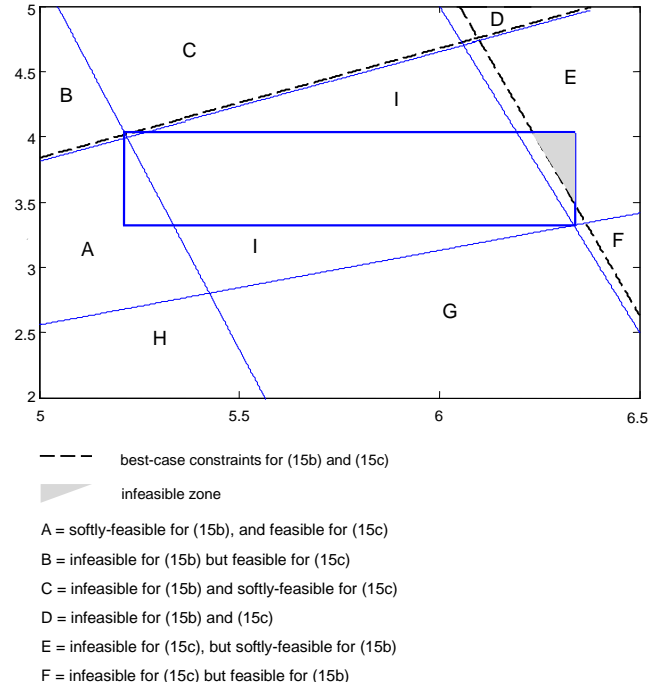

dashed line = best-case constraints for (15b) and (15c)

(shaded) = infeasible zone

A = softly-feasible for (15b), and feasible for (15c)
B = infeasible for (15b) but feasible for (15c)
C = infeasible for (15b) and softly-feasible for (15c)
D = infeasible for (15b) and (15c)
E = infeasible for (15c), but softly-feasible for (15b)
F = infeasible for (15c) but feasible for (15b)

Figure 1. Inexact constraints and solution space obtained through TSM.

dered. Therefore, zones A, I and G would be potentially feasible, which were denoted as 'softly feasible zones" by Huang et al. (2002). In fact, the 'soft' concept corresponds to the existence of uncertainties, demonstrating the existence of a trade-off between system benefit and constraint-violation risk (Huang et al., 2002). A high benefit corresponds to a high risk, and it is impossible to accomplish both high benefit and low risk simultaneously.

Based on the two-step method (TSM) of Huang et al. (1992), model (16) can be solved through formulating the following two submodels:

The first submodel:

$$\text{Max} \quad f^+ = 30x_1^+ - 5.5x_2^- \tag{17a}$$

subject to:

$$8x_1^+ - 14x_2^- \leq 4.2 \tag{17b}$$

$$1x_1^+ + 0.2x_2^- \leq 7 \tag{17c}$$

$$x_1^+ \geq 0, \quad x_2^- \geq 0 \tag{17d}$$

The second submodel:

$$\text{Max} \quad f^- = 26x_1^- - 6.0x_2^+ \tag{18a}$$

subject to:

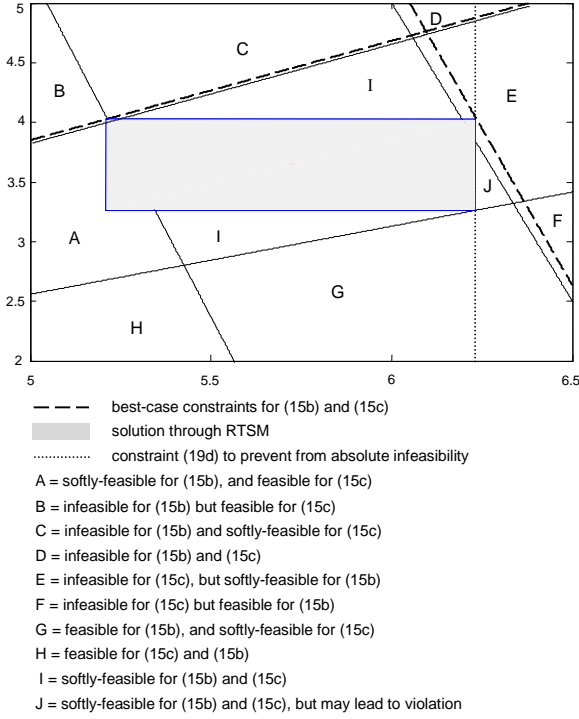$$10x_1^- - 12x_2^+ \leq 3.8 \tag{18b}$$

Figure 2. Inexact constraints and solution space obtained through RTSM.

$$1.1x_1^- + 0.19x_2^+ \leq 6.5 \tag{18c}$$

$$x_1^- \leq x_{1opt}^+, \quad x_2^+ \geq x_{2opt}^- \tag{18d}$$

$$x_1^- \geq 0, \quad x_2^+ \geq 0 \tag{18e}$$

Thus, the solutions of model (16) through TSM are: $x_{1opt}^\pm$ = [5.21, 6.34], $x_{2opt}^\pm$ = [3.32, 4.03], and $f_{opt}^\pm$ = [111.4, 171.8].

The rectangle in Figure 1 shows the solution space obtained through TSM. It is obvious that this space contains a zone where the best-case constraints are violated when decision variables fluctuate within the obtained interval solutions. For example, for the best case of constraint (16c), if $x_1 = x_{1opt}^+$ = 6.34, x_2 = x_{2opt}^+ = 4.03, then we have $1 \times x_1 + 0.19 \times x_2 = 6.34 + 0.19 \times 4.03 = 7.11 > 7$. The shadowed zone in Figure 1 presents the solutions that would violate at least one best-case constraint.

Based on RTSM, model (16) can be transformed into two submodels. The first submodel can be presented as follows:

$$\text{Max} \quad f^- = 26x_1^- - 6.0x_2^+ \tag{19a}$$

subject to:

$$10x_1^- - 12x_2^+ \leq 3.8 \tag{19b}$$

$$1.1x_1^- + 0.19x_2^+ \leq 6.5 \tag{19c}$$

$$x_1^- \geq 0, \quad x_2^+ \geq 0 \tag{19d}$$

The solutions are x_{1opt}^- =5.21, x_{2opt}^+ = 4.03 and f_{opt}^- = 111.38. Thus, the second submodel can be expressed as:

$$\text{Max} \quad f^+ = 30x_1^+ - 5.5x_2^- \tag{20a}$$

subject to:

$$8x_1^+ - 14x_2^- \leq 4.2 \tag{20b}$$

$$1x_1^+ + 0.2x_2^- \leq 7 \tag{20c}$$

$$1x_1^+ + 0.19x_{2opt}^+ \leq 7 \tag{20d}$$

$$x_1^+ \geq x_{1opt}^-, \quad x_2^- \leq x_{2opt}^+ \tag{20e}$$

$$x_1^+ \geq 0, \quad x_2^- \geq 0 \tag{20f}$$

The solutions of submodel (20) are x_{1opt}^+ = 6.23, x_{2opt}^- = 3.26 and f_{opt}^+ =169.1. Consequently, the ultimate solutions of model (16) are: $x_{1opt}^\pm$ = [5.21, 6.23], $x_{2opt}^\pm$ =[3.26, 4.03], and $f_{opt}^\pm$ =[111.38, 169.1].

With the above RTSM, an additional constraint [i.e. constraint (20d) presented as dotted line in Figure 2] is proposed to prevent from violation of the best-case constraint. Through this constraint [i.e. constraint (20d)], zone J is excluded from the solution space. The shadowed zone in Figure 2 shows the ultimate solution space obtained through RTSM. It is indicated that the generated solutions are at least softly feasible in reference to the given constraints (i.e. no absolute violation for constraints).

Model (16) can also be solved through approaches of MILP (Zhou et al., 2008), ThSM-I and ThSM-II (Cao and Huang, 2011). The solutions are presented in Table 1. As shown in Figure 3, the rectangular with solid lines is the solution space obtained through MILP. MILP can ensure no violation of the best-case constraints (present as dash lines in Figure 3). However, it results in a significantly shrunk solution space, leading to serious loss of many feasible decision alternatives. Figures 4 and 5 show the solutions obtained through ThSM-I and ThSM-II. They would not violate the best-case constraints of model (16). However, compared with RTSM, the two approaches would compress the solution space from all dimensions, which may lead to relatively more loss of feasible decision alternatives; it also requires feasibility test which may be redundant for large-scale cases. Particularly when the study cases involve many variables and constraints, the feasibility test may lead to the issue of great computational requirement. In comparison, the solution space acquired through RTSM covers a relatively larger space of the "softly feasible" zone, and can thus avoid significant loss of decision-related information. Also, the procedures of RTSM are not complicated and will not lead to great computational requirement.

Table 1. Solutions for Model (16) through the Previous Solution Methods

	TSM (Huang et al., 1992)	MILP (Zhou et al., 2008)	ThSM-I (Cao and Huang, 2011)	ThSM-II (Cao and Huang, 2011)
			q = 0.833	$q_1 = 0.813$, $q_2 = 1$
$x_{1opt}^{\pm}$	[5.21, 6.34]	[4.57, 6.34]	[5.30, 6.25]	[5.32, 6.23]
$x_{2opt}^{\pm}$	[3.32, 4.03]	[3.32, 3.50]	[3.38, 3.97]	[3.32, 4.03]
$f_{opt}^{\pm}$	[111.4, 171.8]	[98.0, 171.8]	[114.1, 168.78]	[114.0, 168.77]
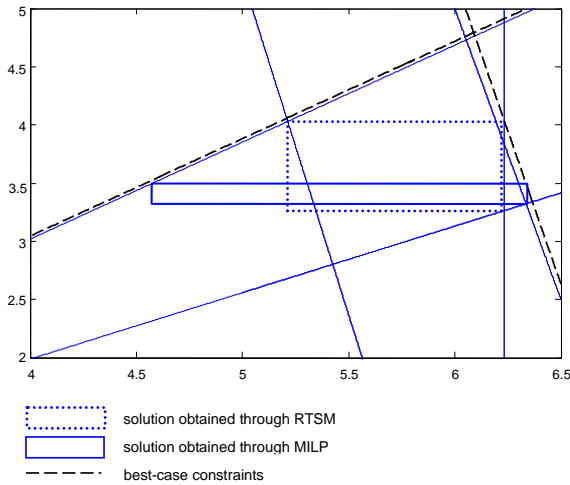
Figure 3. Solutions obtained through MILP and RTSM.
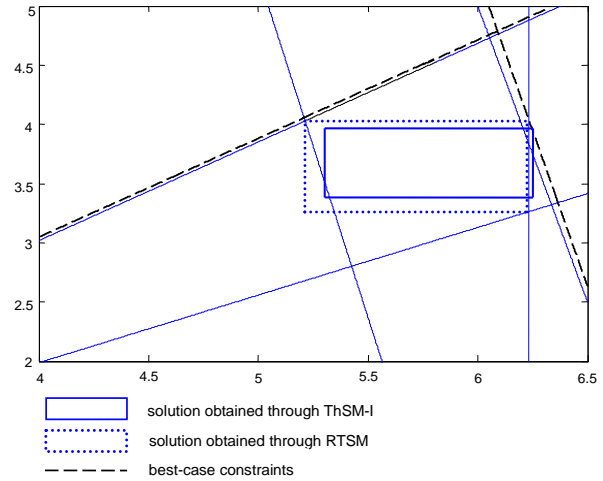
Figure 4. Solutions obtained ThSM-I and RTSM.

5. Conclusions

(1) A robust two-step method (RTSM) has been advanced to solve interval linear programming (ILP) problems. It improves upon the two-step method (TSM) proposed by Huang et al. (1992) through incorporating additional constraints into solution procedures to avoid absolute violation. Compared with TSM, RTSM can guarantee no violation of the best-case constraints when decision variables fluctuated within their solution spaces. RTSM has been applied to a simple case related to environmental management. The results demonstrate applicability of the developed methodology.

(2) Comparisons between the RTSM and the previous solution methods have been undertaken. Different solution spaces have been obtained. It is indicated that: (a) RTSM would generate a relatively larger solution space than other ILP solution methods, and can thus avoid significant loss of decision-related information. (b) In comparison with the three-step method (ThSM), the solution process of RTSM is much simpler. RTSM would not require feasibility test and model constriction. This would be meaningful for its practical applications, since the feasibility test may lead to high computational requirements for problems with a large number of constraints.

(3) Although this study is a new attempt to improve upon the two-step method as proposed by Huang et al. (1992), it is desired that more real-world case studies be undertaken to demonstrate its practical applicability.
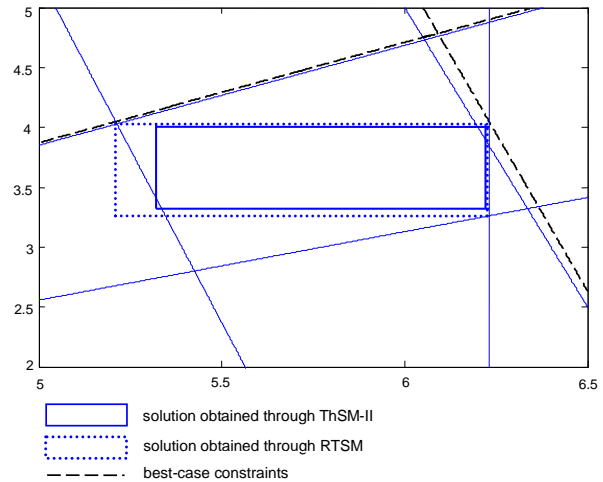

Figure 5. Solutions obtained through ThSM-II and RTSM.

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