Hydrologic Risk Analysis for Nonstationary Streamflow Records under Uncertainty

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Received 7 October 2013; revised 6 April 2014; accepted 6 January 2015; published online 21 September 2015

ABSTRACT. The frequency and magnitude of hydrologic extreme events is critical to water resources management. Traditional hydrologic frequency analysis approaches rely on the inappropriate assumption that hydrology is stationary. To tackle the nonstationarity in the streamflow records, we proposed a hydrologic risk analysis framework for the Xiangxi River, one of the largest tributaries of the Three Gorge Region, China. The year 1989 was identified as the change point of the 50-year flow records through a CUSUM approach combined with a Bootstrap test. Annual peak flow frequency analyses were then carried out for the 50-year time series and the records after the identified change point, respectively. The results revealed that, by taking into consideration nonstationarity, the return period of high peak flood at the Xingshan Station would actually increase. Bayesian inference combined with a MCMC sampling algorithm was also conducted to address uncertainties in parameter estimation and translate them to flow quantile estimates. It was found that the uncertainty in parameter estimation greatly affected the hydrologic design. To better support the associated risk assessment, two risk concepts, the exceedance risk and the occurrence risk, were proposed and analyzed. The results provided important insights into hydrologic nonstationarity and uncertainty, and the proposed framework can provide scientific bases for engineering design and risk management in many other rivers in China and around the world.

Keywords: Nonstationarity, uncertainty, hydrologic frequency analysis, risk assessment, Xiangxi River

1. Introduction

Hydrologic frequency analysis is of great importance to the design and operation of hydraulic infrastructure for streams and rivers (Machado et al., 2015; Sraj et al., 2015). Flow regime analysis can relate the magnitude of extreme events to their frequency of occurrence and thus provide support for the determination of hydrologic design scale (Liu et al., 2015). Traditional hydrologic frequency analysis approaches rely on the attendant stationarity of hydrologic data series (Milly et al., 2008; Gul et al., 2014; Yilmaz and Perera, 2014). However, changing climatic conditions and human disturbances is challenging the assumption of stationarity (Kiang et al., 2011; Jordan et al., 2014; Ma et al., 2014; Madanian et al., 2014). How to reflect the changing probability distribution of hydrologic events and thus address the nonstationarity for hydrologic design and water resources management has been widely studied over the past decades (Ouarda and El-Adlouni, 2011).

The Three Gorges Dam is the largest hydropower station worldwide. It is located on the upstream of the Yangtze River in China, impounding a total area of 59,900 km² (Han et al., 2014a). It has a significant effect on approximately 660 km of the Yangtze River and a total of more than 16 million people. Due to its substantial alteration to the environment of the Yangtze River’s upstream watersheds as well as its significance to China’s economic ascension, many studies have been conducted to investigate the climatic and hydrologic trends in the Three Gorges Region (Xiong and Guo, 2004; Zhang et al., 2006; Mei et al., 2015). Xiong and Guo (2004) analyzed the annual discharge changes of the Yangtze River during 1882 ~ 2001. Zhang et al. (2006) investigated the temporal trends and frequency changes of annual maximum water level and maximum streamflow at three major stations of Yangtze River during the past 130 years. Mei et al. (2015) analyzed the Three Gorges Dam’s effects on downstream hydrological behavior. Many researchers studied the hydrologic alterations resulted from the climate change and human disturbances in this area. However, there were very few studies that focused on their effects on frequency estimates and the associated hydrologic risks. Particularly, most hydraulic infrastructure of the tributaries in the middle reaches of the Yangtze River were built in the 1950s and 1960s, before hydrologic alterations were observed. There are no scientific calculations to support the upgrade of the aging infrastructure and to cope with the changes in the flow regime.

Meanwhile, there are many uncertainties in modeling processes that can affect hydrologic design and risk assessment (Huang et al., 2014; Miao et al., 2014). Parameter uncer-
tainty is a major source of uncertainties in the estimations of flow quantiles and flooding risks (Yang and Yang, 2014; N. Zhang et al., 2014). In the classic frequency analysis approach, distribution parameters are typically estimated using frequentist approaches, such as maximum likelihood estimation (MLE), moment matching estimation (MME) and maximum goodness-of-fit estimation (MGE), where objectively fixed parameters can be generated. The uncertainty of the frequency estimation in the classic process is ultimately dependent on the parameter uncertainty (Huar et al., 2010; Assunaming and Chang, 2014). Compared to traditional approaches, Bayesian methods allow a richer and more complete representation of the uncertainties in flow records (Reis Jr and Stedinger, 2005). They have been proven as effective methods for hydrologic frequency analysis, particularly when the sample size is small (Liang et al., 2012). However, very few studies on the application of Bayesian methods in the Three Gorges Region tributaries have been reported. Estimates of the precision of the flow quantiles, which can be obtained through Bayesian inferences, are desired for a comprehensive uncertainty evaluation and risk assessment in the Three Gorges Region.

Therefore, the objective of this study is to propose a framework for hydrologic frequency analysis and risk assessment with considerations of nonstationarity and uncertainty. The Xiangxi River watershed, a representative watershed of the Three Gorges Region, has been selected to demonstrate the proposed framework. Streamflow statistics of the Xiangxi River during 1961 ~ 2010 will be investigated and a cumulative sum chart approach combined with a Bootstrap test will be used to detect the hydrologic change points and address nonstationarity during the studied period. The 50-year nonstationary streamflow records and the relatively stationary flow records posterior to the detected change point will be analyzed to investigate the effects of nonstationarity. Then Bayesian analysis combined with Markov Chain Monte Carlo sampling will be conducted to generate the posterior distributions of distribution models, flow quantiles and flood risk, and parameters of the Gamma distribution. This study will reflect the nonstationarity and uncertainty in historic flow records and provide robust decision support for hydrologic quantile estimation and the associated risk assessment.

2. Methodology

2.1. Change Point Analysis

The traditional method for hydrologic frequency analysis is based on the assumption that extreme events arise from a stationary distribution (Xiong et al., 2015). However, the assumption of stationarity has been called into question in the recent years (Lee and You, 2013; Ma et al., 2014). Many studies have shown that the hydrologic patterns no longer conform to a stationary and identically distributed random process due to climate change and human interferences (Sang et al., 2010). To reflect the nonstationarity and to identify a relatively stationary recent time series for more reliable hydrologic frequency and risk analysis, a cumulative sum charts (CUSUM) approach combined with a Bootstrap test was adopted for change point analysis.

The CUSUM approach combined with Bootstrap test was first proposed by Taylor (2000). It has been widely used for abrupt change point detection (Smadi and Zghoul, 2006; Renner and Bernhofer, 2011; Tao et al., 2011; Chu et al., 2012). Let \( \{a_i\} = \{a_1, a_2, ..., a_I\} \) represent I data samples of a variable, with the sample mean of \( \bar{a} \) and the sample variance of \( \sigma^2 \). Let the cumulative sum started with \( S_0 = 0 \), then it can be calculated as \( S_i = S_{i-1} + (a_i - \bar{a}) \), \( i = 1, 2, ..., I \). The changes in cumulative sums can help identify shifts in the sample average. For a single change point model, the potential change point \( k^* \) can be identified as the point that returns the maximum cumulative sum, i.e., \( |S_k| = \max \{|S_i|, i = 1, 2, ..., I\} \).

The confidence level of the change point can be determined by conducting bootstrap analysis, which can be performed as below (Tao et al., 2011):

1) Calculate \( S_S = \max(S_i) - \min(S_i), i = 1, 2, ..., I \);
2) Generate a bootstrap sample set of \( I \) samples, presented as \( \{a_i^b\} = \{a_1^b, a_2^b, ..., a_I^b\} \);
3) Calculate the cumulative sums of the bootstrap samples (i.e., \( S_b^S = S_{b,1}^S + (a_i^b - \bar{a}_b^S), i = 1, 2, ..., I \) ), as well as the corresponding \( S_b^S \);
4) Compare the values of \( S_S \) and \( S_b^S \);
5) Iterate Steps 2) to 4) for \( N \) times;
6) Record \( n \) as the number of bootstraps where \( S_S < S_b^S \); and
7) The confidence level (CL) of the identified change point can be calculated as \( CL = n / N \times 100\% \).

2.2. Hydrologic Frequency Analysis Model

The classic hydrologic frequency analysis approach is to determine a distribution function to fit the observation data. Identifying the type of distribution is crucial for calculating the nonexceedance probability that can later be used as the inputs of hydrologic risk assessment (Gebregiorgis and Hossein, 2012). Generalized extreme value (GEV), Gamma, Pearson Type III (P-III), and Log-normal distributions are commonly adopted distribution functions for fitting the annual peak flow series (Wang et al., 2001; Sudheer et al., 2003; Q. Zhang et al., 2014). In this study, the Gamma distribution was chosen because it was widely applied to hydrologic frequency analysis and it was recommended by the Chinese Ministry of Water Resources (Yue et al., 2001; Liu et al., 2011).

Suppose \( x_t, x_s, ..., x_c \) is a time series of annual peak flow. For a large \( n \), we have:

\[
\Pr\{x_s \leq z\} \approx H(z)
\]

(1)

where:

\[
H(z) = \int_0^z \frac{1}{\Gamma(a)b^a} x^{a-1} e^{-\frac{x}{b}} dx, a > 1
\]

(2)

\( H(z) \) is called the Gamma model, where \( a \) and \( b \) are the
shape and scale parameters, respectively. It can be used to fit the observation data of annual peak flow for hydrologic frequency analysis.

2.3. Bayes’ Theorem

Bayesian inference provides an alternative for parameter estimation, which allows probability to represent subjective uncertainty or subjective belief (Eshky, 2008). In the Bayes’ theorem, parameters are treated as random variables and the corresponding likelihood is described with probability density functions (PDF) (Sang et al., 2010). The PDF can be obtained by starting with a prior distribution and then converting it into a posterior distribution through the inclusion of additional information provided by observation data. Since additional observation is added to the prior knowledge available, the posterior distribution can give a more complete representation of the parameter space. The Bayes’ theorem can be outlined as follows:

\[ P(\theta|x) = \frac{p(x|\theta)P(\theta)}{\int p(x|\theta)P(\theta)d\theta} \]  

(3)

where parameter \( \theta \) is the parameter to be estimated, \( P(\theta) \) is the PDF of the prior distribution for \( \theta \), \( p(x|\theta) \) is the likelihood function, \( \Theta \) is the parameter space of \( \theta \), and \( P(\theta|x) \) is the PDF of the posterior distribution for \( \theta \).

To estimate the PDFs of the posterior distributions for parameters \( a \) and \( b \) in the Gamma model, the prior distributions were first defined as normal distributions based on the knowledge obtained from MLE, MME and MGE. The prior distributions can be given as follows:

\[ P(a) = \frac{1}{\sqrt{2\pi} \sigma_a} e^{-\frac{(a-a_0)^2}{2 \sigma_a^2}} \]  

(4)

\[ P(b) = \frac{1}{\sqrt{2\pi} \sigma_b} e^{-\frac{(b-b_0)^2}{2 \sigma_b^2}} \]  

(5)

where \( a_0 \) and \( b_0 \) are the means of \( a \) and \( b \)’s prior distributions, respectively. Then the likelihood function is given by:

\[ p(x|\theta) = \prod_{i=1}^{n} p(x_i|\theta) = \prod_{i=1}^{n} \frac{1}{b^\alpha} \int_{0}^{\infty} x^{a-1} e^{-\frac{x}{b}} dx \]  

(6)

Accordingly, the equation of the posterior distribution can be obtained by substituting Equations 4 to 6 into Equation 3. In this study, instead of solving the equation analytically, an empirical estimate of the posterior distribution was generated using statistical inferences based on a Markov Chain Monte Carlo (MCMC) technique.

2.4. Metropolis-Hastings Algorithm

The MCMC approach with Metropolis-Hastings (MH) steps were adopted to simulate a Markov chain with equilibrium distribution of the posterior distribution of the target parameter \( \theta \), named \( \pi(\theta) \) (Kastner et al., 2013). An MH algorithm can be summarized as follows:

1) Set an initial parameter value \( \theta_0 \);
2) Identify a proposal function \( q(\theta_{-1} \rightarrow \theta') \), where \( \theta_{-1} \) is the current state of the chain, and \( \theta' \) is the new state;
3) Propose a new parameter value \( \theta' \) based on \( \theta_{-1} \) and the probability density function;
4) Compute the acceptance probability \( a(\theta' | \theta_{-1}) = \min(1, \frac{\pi(\theta')q(\theta_{-1} \rightarrow \theta')}{\pi(\theta_{-1})q(\theta' \rightarrow \theta_{-1})}) \);
5) Draw a random number \( C \) from the uniform distribution \( U(0,1) \), and compare the \( C \) value with the acceptance probability obtained in Step 4. If \( a(\theta' | \theta_{-1}) > C \), accept the proposed value and let \( \theta_i = \theta' \), otherwise reject \( \theta' \) and let \( \theta_i = \theta_{i-1} \);
6) Iterate Steps 2) to 5) to generate more samples for the chain.

In the chain, the stochastic properties of \( \theta \) are independent of the previous states \( \theta_1, \theta_2, ..., \theta_{i-1} \) (Hao et al., 2015). The posterior distributions of parameters \( a \) and \( b \) were obtained by running the MH algorithm twice separately.

2.5. Risk and Return Period Analysis

Hydrologic risk in general is defined as the exposure to an extreme, dangerous, hazardous or undesired event (USA-CE, 1988). It is measured by probability and it can be estimated by analyzing historical flow data (USACE, 1988). This study focused only on risks related to hydrologic processes in terms of annual peak flow. According to Equations 1 and 2, the nonexceedance probability of a certain flow value \( z \) can be given by the cumulative distribution function (CDF) of the Gamma distribution that fits the historic annual peak flow data:

\[ q = Pr(x < z) = F(z) = \int_{0}^{x} \frac{1}{b^\alpha} \int_{0}^{\infty} x^{a-1} e^{-\frac{x}{b}} dx \]  

(7)

Return period is an important concept derived from the nonexceedance probability. It can be calculated as follows (Salas and Obeysekera, 2014):

\[ T = (1-q)^{-1} \]  

(8)

Even though a flow with a return period of \( n \) years does not mean the maximum flow that is likely to occur during the \( n \) years, it is still an important criterion in hydrologic and en-
environmental engineering design practice. With the probabilistic estimates regarding $q$ provided by Bayesian inference, the exceedance risk that the flow volume with a return period of $T$ exceeds $q$ can be defined as follows:

$$R_e = 1 - Pr\{q_T\}$$  \hspace{1cm} (9)

where $Pr\{q_T\}$ can be calculated according to the sample set of $q$ by drawing random values from the posterior distributions of parameters $a$ and $b$ and substituting them to Equation 7. Determination of the exceedance risk is useful for many engineering practices, such as the design of hydraulic infrastructure and the development of hydrologic risk management projects.

Furthermore, another type of risk, which is the occurrence risk $R_o$ defined as the probability of the occurrence of a flow that exceeds $z$ in $n$ years, can be given as follows (Gebregiorgis and Hossain, 2012):

$$R_o = 1 - q^n = 1 - (1 - 1/T)^n$$  \hspace{1cm} (10)

3. Study Area and Data Analysis

The Xiangxi River watershed is located in Hubei Province, China (Figure 1). The Xiangxi River is approximately 94 km in length from its source in the Shennongjia Forestry District to the mouth at Zigui County, draining an area of 3,200 km$^2$ into the Yangtze River. It is in the vicinity of the Three Gorges Dam, the largest operating hydropower facility over the world. The Xiangxi River watershed is a representative watershed of the Three Gorges areas in the middle reach of the Yangtze River. It has a typical subtropical continental monsoon climate, with a mean annual temperature of 17 °C and an annual precipitation of 900 ~ 1,200 mm (Li et al., 2015). Precipitation in this area is more intense in summer than in winter. Approximately 70% of the precipitation received between May and September, is rainfall. Over 80% of the area is mountainous, and the land cover is dominated by mixed needle-leaf and broad-leaf forests (Han et al., 2014b; Liu et al., 2014).

Due to abundant hydropower and mineral resources in the Xiangxi River Watershed, the local economy experienced a rapid growth during the 1980s and 1990s (Li et al., 2013). More than 50 hydropower stations and a number of reservoirs, including two cascade reservoirs (i.e., Gudongkou I Reservoir and Gudongkou II Reservoir), were built. These intensive human activities posed profound impacts on the hydrological cycle, which led to significant changes in evapotranspiration, precipitation and streamflow in the past decades (Seeber et al., 2010; Han et al., 2014a). The changes in precipitation and streamflow trends usually imply that there would be flood risk changes supervened (Obeysekera and Salas, 2013; Liu et al., 2014). Therefore, it is desired to re-evaluate the associated risks under changing hydrologic conditions, to provide reliable decision support for water resources management and flood control.
4. Result Analysis and Discussion

4.1. Change Point Analysis

Streamflow statistics for each year during 1961 ~ 2010 were calculated and the corresponding time series was generated for change point analysis. The 29th point, i.e., 1989, was identified as the most probable change point in terms of twelve streamflow statistics including Qmean, Q10, Q30, Q50, Q70, Q90, Q7-day low, Q7-day mean, Q7-day high, Q14-day low, Q14-day mean, Q14-day high. The confidence levels of the tests are given in Table 1. The change point of Qmean, Q10, Q30, Q7-day mean, Q7-day low, Q7-day high and Q14-day low are at an acceptable significance level. Even though high confidence level values were not reached during the bootstrap test, 1989 is still the point that returns the maximum cumulative sums of the other streamflow statistics. It is implied that significant changes of the hydrologic time series occurred since 1989, and thus 1989 can be considered as the change point of the hydrological time series (Han et al., 2014a).

Table 1. Confidence Level of the Change Point Tests

<table>
<thead>
<tr>
<th>Streamflow statistic</th>
<th>Confidence level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qmean</td>
<td>0.90</td>
</tr>
<tr>
<td>Q10</td>
<td>0.99</td>
</tr>
<tr>
<td>Q30</td>
<td>0.94</td>
</tr>
<tr>
<td>Q50</td>
<td>0.58</td>
</tr>
<tr>
<td>Q70</td>
<td>0.77</td>
</tr>
<tr>
<td>Q90</td>
<td>0.89</td>
</tr>
<tr>
<td>Q7-day low</td>
<td>0.80</td>
</tr>
<tr>
<td>Q7-day mean</td>
<td>0.99</td>
</tr>
<tr>
<td>Q7-day high</td>
<td>0.94</td>
</tr>
<tr>
<td>Q14-day low</td>
<td>0.90</td>
</tr>
<tr>
<td>Q14-day mean</td>
<td>0.89</td>
</tr>
<tr>
<td>Q14-day high</td>
<td>0.60</td>
</tr>
</tbody>
</table>

To illustrate the hydrological changes, times series of Qmean, Q10, Q50, and Q90 are given in Figure 2. There is a general decreasing tendency of the four streamflow-related statistics. The hydrological alterations might have resulted from the unavoidable effects of climate change and intensive human interferences in recent decades. Variations in the intrannual precipitation distribution, which might affect the streamflow regime in the Xiangxi River Watershed, have been found in previous studies (Zhang et al., 2011). Decrease of the streamflow statistics might also be attributed to the significant decreasing trends in the evapotranspiration of the Yangtze River basin (Xu et al., 2006). In addition, the hydropower resources of the Xiangxi River were extensively exploited in the 1990s. Over 50 hydropower stations were constructed within the watershed (Guo et al., 2000; Wu et al., 2009). A large portion of the streamflow was diverted for the purpose of electricity generation, significantly affecting streamflow (Wu et al., 2007). Land use changes due to land cultivation and mining activities were also found during the 1990s (Seeber et al., 2010; Han et al., 2014a; Li et al., 2014).

4.2. Frequency Analysis for Nonstationary Flow Records

The hydrological alterations could lead to evident changes in the probability behavior of the peak flow series. In order to reflect the effects of nonstationarity, frequency analysis of the peak flow records with and without the detected change point was conducted (Figure 3). The results demonstrates that cumulative probabilities of the peak flow values increase remarkably in the posterior change point time series. The probability changes of the middle-level peak flow values are the most significant. For instance, the cumulative probability of an annual peak flow of 477 m^3/s is 0.56 in the 50-year time series, and it rises to 0.68 in the posterior change point time series with an increase of 21.75%. The cumulative probability increases of the 438 and 456 m^3/s flow records are...
as high as 23.11 and 22.38%, respectively. It is obvious that the Gamma distribution that fitted the 50-year peak flow time series was no longer acceptable for describing the probability behavior of the series after the change point. Correspondingly, the designed peak flow values needed to be re-calculated. The results indicate that, taking into account the nonstationarities, there is an increase in return periods of floods. For instance, when the 50-year peak flow time series was considered as stationary, the return period of a 1,000 m$^3$/s peak flow rate was calculated as 25 years and that of a 1,200 m$^3$/s peak flow rate was 84 years. However, when the hydrologic design was based on the time series posterior to the change point, the return periods of the 1,000 m$^3$/s and peak flow rates were calculated as 34 and 116 years, respectively.

Frequency analyses were further conducted with regard to the annual peak flow series posterior to the change point. Three parameter estimation methods, i.e., MLE, MME and MGE, were applied to estimate the parameters of the Gamma distribution. The obtained parameters estimation and calculated peak flow values in different return periods are presented in Table 2. It indicates that different estimation methods resulted in varied parameter values. However, the different parameterizations generated very similar modeling results of the nonexceedance probability. The differences among the estimators are relatively minor. The $R^2$ value of the three models are 0.981, 0.981 and 0.987, respectively. This is called equifinality in parameterization, which has been found in many empirical studies (Zak and Beven, 1999; Beven, 2006; Tang and Zhuang, 2008). Even though similarly good fits to the nonexceedance probabilities of the historic peak flow were obtained, the hydrologic design values in the 10-year, 50-year, 100-year and 200-year return period would vary.

### 4.3. Bayesian Parameter Estimation

A total of 10,000 pairs of $a$ and $b$ samples were obtained using the Bayes' theorem and the MCMC-MH algorithm described in Section 2. Visual inspection of the chains was conducted to determine when convergence is achieved (Hao et al., 2015). It was found that the two chains mix well during the latter 8,000 iterations, and thus the 8,000 samples were used to generate the posterior distributions of Parameters $a$ and $b$ for the peak flow records at the Xingshan Station. As shown in Figure 4a, the mean values, the center values and the detailed shapes of parameter $a$'s posterior distributions under nonstationarity and stationarity are different. As for parameter $b$, even though the mean value under stationarity is very close to that under nonstationarity, the most probable values under the two assumptions do not coincide with each other. The joint probability distribution of $a$ and $b$ are presented in Figure 4b. The distribution is a complicated mixture distribution, and it is widely spread with two distinct peaks. It is obvious that these uncertainties may affect the estimated quantiles and probabilistic forecast results of hydrologic design results. Thus, it

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**Table 2. Frequency Analysis Results Based on Maximum Likelihood Estimation, Moment Matching Estimation, and Maximum Goodness-of-fit Estimation**

<table>
<thead>
<tr>
<th>Method of parameter estimation</th>
<th>Estimated parameters</th>
<th>Goodness-of-fit</th>
<th>Peak flow in different return periods</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shape</td>
<td>Scale</td>
<td>$R^2$</td>
</tr>
<tr>
<td>MLE</td>
<td>4.028</td>
<td>116.573</td>
<td>0.981</td>
</tr>
<tr>
<td>MME</td>
<td>3.934</td>
<td>119.366</td>
<td>0.981</td>
</tr>
<tr>
<td>MGE</td>
<td>5.030</td>
<td>89.479</td>
<td>0.987</td>
</tr>
</tbody>
</table>

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**Figure 3. Distributions and return periods of flow under stationarity and nonstationarity.**
is crucial to include parameter uncertainty in hydrologic frequency analysis and risk assessment.

4.4. Frequency Analysis and Hydrologic Risk Under Uncertainty

The parameter uncertainty was translated into uncertainty on hydrologic calculation results using an approach revised from the work of Coles et al. (2003). Two thousand pairs of $a$ and $b$ samples, denoted as $\theta_m$, where $\theta_m = \{a_m, b_m\}$, $m = 1, 2, \ldots, 2000$, were first drawn from their posterior distributions using MCMC sampling. Then the peak flow quantile $q_{m,T}$, flow $q_{m,T}$ with return period $T$, exceedance risk $R_{E,m}$ and occurrence risk $R_{O,m}$ were computed for each parameter sample $\theta_m$. Finally, the samples of $q_{m,T}$, $q_{m,T}$, $R_{E,m}$ and $R_{O,m}$ were histogrammed to yield the corresponding distributions (Coles et al., 2003; Huard et al., 2010).

By making probabilistic $a$ and $b$ the two parameters driving the Gamma distribution model, the uncertainty in the non-exceedance probability of a certain flow volume were first quantified. The mean value and 95% confidence interval are presented in Figure 5. The range of the 95% confidence interval is the largest for the flow volume of $[370, 480]$ m$^3$/s, where the diameter of the interval is higher than 0.2. The 95% peak flow quantile with a confidence level of 95% is approximately $[840, 1,000]$ m$^3$/s. Furthermore, the 95% confidence interval of the flow volume with a return period $T$ were obtained, as given in Figure 6a. It is indicated that the longer the return period, the higher the designed peak flow. It is also implied that the parameter uncertainty as well as the probabilistic cumulative probability estimate would lead to uncertainties in designed peak flow volume. The uncertainty would be more significant for the flow volume with a longer return period. For instance, the 95% confidence interval of the peak flow with a return period of 100 years would range from 1,081 to 1,274 m$^3$/s; that of the peak flow with a return period of 1,000 years would range from 1,416 to 1,644 m$^3$/s. Meanwhile, the uncertainties in the return period estimation of high flow volume would be enormous. For instance, the return period of a peak flow of 1,400 m$^3$/s would vary from 200 years to as long as 1,000 years, not to mention the peak flow of 1,500 m$^3$/s or higher. To reflect the uncertainty in the estimation of the designed values, the probability that the flow with a return period of $T$ exceeds a certain value is proposed as the exceedance risk. The exceedance risk of flow $q_T$ with a return period of $T$ is illustrated in Figure 6b.
The occurrence risk of flooding depends on the designed life time of the dam or hydraulic structure, which is usually expected to be 100 years or longer (Nagy et al., 2013). Therefore, occurrence risk of flood at the Xingshan Station for a designed period of 100 years was analyzed. Figure 7a demonstrates that the higher designed flood value, the lower the risk. The designed flood of 1,500 m$^3$/s takes the occurrence risk of 13.2%. With the probabilistic peak flow quantiles generated by Bayesian inference, the confidence levels of the occurrence risk were further obtained as shown in Figures 7a and 7b. The 95% confidence interval of occurrence risk for the designed flow of 1,500 m$^3$/s is 5.2 to 22.0%. It is difficult to determine an exact acceptable threshold for the occurrence risk. However, in engineering practice, it is recommended to keep the risks as low as possible for the purpose of ensuring safety and maintaining economic feasibility (Gebregiorgis and Hossain, 2012). The occurrence risk and its probability distribution can help the designer minimize the associated risks.

5. Conclusions

This study proposed a framework for hydrologic frequency analysis and risk assessment with consideration to both nonstationarity and uncertainty. The proposed approach was applied to the Xiangxi River in China. Nonstationarity analysis was first conducted through a CUSUM approach combined with Bootstrap test. The year 1989 was identified as the change point of the 50-year $Q_{\text{mean}}$, $Q_{10}$, $Q_{30}$, $Q_{7 \text{-day mean}}$, $Q_{7 \text{-day low}}$, $Q_{7 \text{-day high}}$ and $Q_{14 \text{-day low}}$ time series with acceptable significance level. The annual peak flow frequency analyses were then carried out for the 50-year time series and the records after the identified change point, respectively. The results indicated that, the Gamma model that fitted the 50-year peak flow time series was not acceptable for describing the probability behavior of the series after the change point. It was also revealed that when taking nonstationarity into conside-
rati on, the return period of high peak flood at the Xingshan Station would actually increase, which should be considered for future hydrologic design.

Furthermore, uncertainty analysis regarding the flow record posterior to the change point was conducted based on Bayesian inference and MCMC sampling. It was found that the uncertainty in parameter estimation greatly affected the estimation of the hydrologic design values. The effects on the estimated return periods of high flow volumes were particularly significant. In addition, two risk concepts were proposed to support hydrologic risk assessment. The exceedance risk was defined as the probability that the flow with a return period of 7 years exceeds a certain volume, and the occurrence risk was defined as the probability that a flow high than \(z\) occurs in a \(n\)-year period. The results provided important insights into the hydrologic nonstationarity and uncertainty of the Xiangxi River. They also provided scientific bases for many other rivers in China and around the world, to support water managers. The proposed approaches are generic and robust flood frequency analysis and risk assessment for local water managers. The proposed approaches are generic and robust flood frequency analysis and risk assessment for local water managers.

### Acknowledgment

This research was supported by the Natural Sciences Foundation (51190095, 51520105013), the 111 Project (B140-08), and Natural Sciences and Engineering Research Council of Canada.

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