Supporting Information

A Novel Fuzzy Best-Worst Multicriteria Decision-Making Method Based on the Dual Interval Algorithm for Environmental Decision Support Systems

Y. Cheng[1 *], L. Jin[1 * **], H. Y. Fu[2 **], Y. R. Fan[3], R. L. Bai[1], and Y. Wei[1]

[1] College of Environmental Science and Engineering, Xiamen University of Technology, Xiamen, Fujian 361024, China
[2] Key Laboratory of Environmental Biotechnology (XMUT), Fujian Engineering and Research Center of Rural SewageTreatment and Water Safety, Xiamen University of Technology, Xiamen, Fujian 361024, China
[3] Department of Civil and Environmental Engineering, Brunel University London, Uxbridge, Middlesex UB8 3PH, UK

Analysis of FBWM Methods

Fuzzy numbers are an extension of exact numbers and be able to express the uncertainty embedded in realistic data. The FBWM method uses fuzzy numbers $\tilde{A}_{Bi} = \{\tilde{a}_{B1}, \tilde{a}_{B2}, \ldots, \tilde{a}_{Bn}\}$ to represent the degree of preference between criteria, which makes the construction of the problem model more realistic. However, there are some drawbacks in the FBWM algorithm proposed by Guo and Zhao (2017) as well as Dong et al. (2021), described as follows:

(1) Guo and Zhao (2017) proposed in [a] to use the upper boundary value "a_{Bj}^{u} "of fuzzy preference values $\tilde{a}_{Bi} = (a_{Bj}^{l}, a_{Bj}^{m}, a_{Bj}^{u})$ to calculate the fuzzy consistency ratio, which is a correct calculation idea. However, upon inspection, we discovered that the minimum consistent value result is inconsistent with theory. The modified fuzzy consistency catalog is discussed in the main text.

(2) In the solution of Equation (8), Guo and Zhao (2017) used a limited fuzzy subtraction operation, which may produce unreliable results. The suitable fuzzy subtraction operation is described as the following definition:

Definition. Let $\tilde{A} = (a_l, a_m, a_u)$ and $\tilde{B} = (b_l, b_m, b_u)$ be two triangular fuzzy number, then we get $\tilde{A} - \tilde{B} = (a_l - b_u, a_m - b_m, a_u - b_l)$.

Additionally, the process of solving Equation (8) involves the sorting operation of fuzzy numbers. In Guo's fuzzy algorithm, if "$\tilde{A} < \tilde{B}$" then "$a_l < b_l \&\& a_m < b_m \&\& a_u < b_u$", which gave the local solution set of the fuzzy sorting algorithm but ignored other possible solutions. The new algorithm proposed in this paper avoids this problem (described in section 3).

(3) Expanding Equation (8) yields as follows:

$$\tilde{\omega}_B - \tilde{a}_{Bj}\tilde{\omega}_j \le \tilde{\xi} \cdot \tilde{\omega}_j \&\& \tilde{\omega}_B - \tilde{a}_{Bj}\tilde{\omega}_j \ge -\tilde{\xi} \cdot \tilde{\omega}_j \tag{S1}$$

where the "$\tilde{\xi} \cdot \tilde{\omega}_i$" is a non-linear term. To linearize the nonlinear FBWM algorithm, Dong et al. (2021) used Equation (9) to replace Equation (S1). In this process, "$\tilde{\xi} \cdot \tilde{\omega}_i$" are deleted. This causes a computational bias in the later calculation process because "$\tilde{\xi} \cdot \tilde{\omega}_i$" is not having an effect on the calculation results when the variable $\tilde{\omega}_j$ changes.

(4) Dong et al. (2021) directly used fuzzy numbers to de-rive the maximum inconsistency (CI), which is incorrect. By combining Definition 3 and Equation (5), we can learn that "CI = ξ_i" represents the maximum inconsistency of a set of criteria weights. Clearly, the maximum possible inconsistency should be deduced by $max\{ a_{BW}^{t}, t \in (l, m, n)\}$ since the crisp number in the initial setting $A_{Bj} = (a_{B1}, a_{B2}, \ldots, a_{Bn})$ be replaced by fuzzy numbers.

From the above discussion, it can be learned that it is not reasonable to use fuzzy quadratic operations directly in the process of solving the FBWM problem. Firstly, the BWM problem is a nonlinear problem, and secondly, the multiplication and division operations of fuzzy numbers are not a simple extension of which in crisp number operation. The proof process is as follows:

Proof. Both FBWM algorithms use "$(a_l, a_m, a_u) \times (b_l, b_m, b_u) = (a_l b_l, a_m b_m, a_u b_u)$" as a multiplication operation for $\tilde{A} \times \tilde{B}$. Let $\mu(\hat{C}) = (a_l b_l, a_m b_m, a_u b_u)$ be the membership function of fuzzy number C, and set a value $\mu_0 \in [0, 1]$, we get:

$$\mu(a_1) = \mu(a_2) = \mu(b_1) = \mu(b_2) = \mu_0 \Rightarrow \mu(a_1 b_1) = \mu(a_1 b_2) = \mu(a_2 b_1) = \mu(a_2 b_2) = \mu_0 \tag{S2}$$

Let $c_1 = a_1 b_1,\ c_2 = a_1 b_2,\ c_3 = a_2 b_1,\ c_4 = a_2 b_2$, then we have:

$$\mu(c_1) = \mu(c_2) = \mu(c_3) = \mu(c_4) = \mu_0 \tag{S3}$$

Now we get four values with the same degree of membership function, then it is clear that the membership function of $\hat{C}$ cannot be in a triangular form. This violates the conclusion $\mu(\hat{C}) = (a_l b_l, a_m b_m, a_u b_u)$ in the above proposition (which has a triangular fuzzy solution).

Furthermore, both FBWM methods obtained incorrect consistency ratios. Guo and Zhao (2017) used the correct calculation method but made an error in the calculation process, while Dong et al. (2021) used the wrong calculation method to solve for the fuzzy consistency ratio.

Therefore, to extend the initial values and weights of the criteria preferences in the BWM method from crisp numbers to fuzzy numbers, directly performing various arithmetic operations on fuzzy numbers should be avoided, and the solution procedure of fuzzy consistency ratio also needs to be revised. How to transfer the fuzzy information contained in $\tilde{A}_B = [\tilde{a}_{B1}, \tilde{a}_{B2}, \ldots, \tilde{a}_{Bn}]$ and $\tilde{A}_W = [\tilde{a}_{1W}, \tilde{a}_{2W}, \ldots, \tilde{a}_{nW}]$ to the solution set $\tilde{\omega} = (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n)$ without directly performing fuzzy arithmetic operations on the fuzzy weights has become a new proposition. To solve this question, we propose a novel and appropriate FBWM method based on a dual interval algorithm in section 3.

Table S1. Different Value of ω_1, ω_2, ω_3 and $R(\xi^*)$ for $a_{32}, a_{21} \in \{WI, FI, VI, AI\}$

Linguistic terms	a_{32}	*WI*	*FI*	*VI*	*AI*
a_{21}		(2/3, 1, 3/2)	(3/2, 2, 5/2)	(5/2, 3, 7/2)	(7/2, 4, 9/2)
	$R(\xi^*)$	0.1718	0.0990	0.0440	0.0000
WI	ω_1	(0.143, 0.167, 0.193)	(0.143, 0.167, 0.194)	(0.143, 0.167, 0.194)	(0.143, 0.167, 0.194)
	ω_2	(0.286, 0.299, 0.309)	(0.235, 0.242, 0.256)	(0.196, 0.199, 0.207)	(0.166, 0.167, 0.171)
	ω_3	(0.496, 0.535, 0.571)	(0.551, 0.592, 0.622)	(0.599, 0.635, 0.661)	(0.635, 0.667, 0.691)
	$R(\xi^*)$	0.0990	0.0150	0.0530	0.0980
FI	ω_1	(0.125, 0.143, 0.167)	(0.125, 0.143, 0.167)	(0.125, 0.143, 0.167)	(0.125, 0.143, 0.167)
	ω_2	(0.330, 0.349, 0.359)	(0.274, 0.286, 0.299)	(0.229, 0.235, 0.241)	(0.193, 0.196, 0.199)
	ω_3	(0.474, 0.507, 0.544)	(0.535, 0.571, 0.601)	(0.592, 0.622, 0.646)	(0.635, 0.661, 0.682)
	$R(\xi^*)$	0.0400	0.056	0.1420	0.2040
VI	ω_1	(0.111, 0.125, 0.143)	(0.111, 0.125, 0.143)	(0.111, 0.125, 0.143)	(0.111, 0.125, 0.143)
	ω_2	(0.375, 0.399, 0.414)	(0.314, 0.331, 0.349)	(0.264, 0.274, 0.286)	(0.222, 0.229, 0.235)
	ω_3	(0.443, 0.476, 0.514)	(0.507, 0.544, 0.575)	(0.571, 0.601, 0.625)	(0.622, 0.646, 0.667)
	$R(\xi^*)$	0.0025	0.1024	0.2063	0.2902
AI	ω_1	(0.100, 0.111, 0.125)	(0.100, 0.111, 0.125)	(0.100, 0.111, 0.125)	(0.100, 0.111, 0.125)
	ω_2	(0.416, 0.444, 0.465)	(0.354, 0.375, 0.399)	(0.300, 0.314, 0.331)	(0.254, 0.264, 0.274)
	ω_3	(0.410, 0.444, 0.484)	(0.476, 0.514, 0.546)	(0.544, 0.575, 0.600)	(0.601, 0.625, 0.646)

References

Dong, J.Y., Wan, S.P. and Chen, S.M. (2021). Fuzzy best-worst method based on triangular fuzzy numbers for multi-criteria decision-making. *Inf. Sci*. 547, 1080-1104. https://doi.org/10.1016/j.ins. 2020. 09.014

Guo, S. and Zhao, H.R. (2017). Fuzzy best-worst multi-criteria decision-making method and its applications. *Knowledge-Based Syst.* 121, 23-31. https://doi.org/10.1016/j.knosys.2017.01.010