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Impact of Carbon Emissions and Advance Payment on Optimal Decisions for Perishable Products via Parametric Approach of Interval

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ABSTRACT. Under the heading of sustainability challenges, this study incorporates payment procedures and inventory selections. The article's goal is to acquire insight into how the methods of payment affect perishable product inventory selections under the widely utilised carbon tax regime in view of policies to reduce emissions. Uncertainty arises as a natural consequence of the unpredictable behaviour of customers. Keeping these impacts in mind, an interval-valued inventory model is introduced, where all the related inventory parameters are considered interval-valued. In addition, the vendor offers an interval-valued discount rate to customers against advance payment. Due to the inventory parameters being chosen as interval-valued, the objective function is changed into an interval-valued form, and the corresponding differential equation is changed into an interval differential equation. To solve the interval-valued differential equation, a parametric approach to interval is introduced, and the corresponding interval-valued objective function is constructed. Interval order relations and the MATHEMATICA software are used to solve the objective function with interval values. To assess the validity of the proposed model, one numerical example is solved, and a sensitivity analysis of the ideal course of action is performed.

Keywords: interval differential equation, interval-valued deterioration, interval-valued discount, interval-valued carbon emission, environmental regulations, interval-valued inventory

1. Introduction

An interval-valued discount rate against an advance payment refers to a payment arrangement where a discount rate is applied to the interval-valued advance payment made by the customer or buyer. The discount rate is based on the concept of time being worth money, and it represents the opportunity cost of not having the full payment amount available to the buyer at the time of the payment in advance. The interval-valued discount rate against advance payments can benefit both the buyer and the seller. The buyer can save money by taking advantage of the discount rate, while the seller can benefit from having a portion of the advance payment and the assurance that the buyer is committed to completing the transaction. However, it is important to note that the discount rate depends on a number of criteria, including the length of the payment interval, the creditworthiness of the buyer, and the risk associated with the transaction. Therefore, it is essential to have a clear agreement in place that outlines the terms and conditions of the payment arrangement to avoid misunderstandings or disputes down the line. In the literature on inventory, a cash payment is used to create the conventional economic order quantity (EOQ) model by Harris (1990). The cash-on-delivery model for perishable items was then investigated by Feng et al. (2017) when the customer demand rate varies based on selling price, expiration date, and displayed stocks. Chen (2018) examined the production inventory model choices for a system with multiple retailers for a single manufacturer of perishable goods with a cash payment. In order to save money and time, Zhang (1996) developed an ideal payment system for paying small-amount invoices in advance. This advance payment was extended by Teng et al. (2016) to include perishable goods with expiration dates. For evaporating products, Taleizadeh (2014) presented an inventory model with a cash-advance payment (i.e., some in advance and the balance in cash). Moreover, Zhang et al. (2014) established an EOQ model in which the vendor provides a price reduction in exchange for an advance-credit payment. Chang et al. (2019), Wu et al. (2018), and Li et al. (2017) generalised the previously stated model under the cash-advance-credit payment scheme. Khan et al. (2019) resolved a perishable goods inventory issue by considering partial backlogs and an advance payment. Later, Khan et al. (2020) investigated the impact of discounts on supply decisions for a deteriorating good, taking into account an advance payment. Rahman et al. (2021) developed a perishable inventory model with advance payment considering a hybrid stock and price-dependent demand rate. Duary et al. (2022) for-

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mulated a forecasting model for inventories of decaying goods with installment opportunities to make payments in advance. Recently, Khan et al. (2023a) solved an advance payment-based inventory model with backlogged shortages under a variable payment installment.

Deteriorative products are goods that have a limited shelf life and will eventually degrade in quality or spoil if not used or consumed within a certain time frame. This can include perishable food items such as fresh produce, meat, and dairy products, as well as certain chemicals and pharmaceuticals. The degradation of these products can occur through various mechanisms, such as oxidation, microbial growth, or chemical reactions, which can affect their taste, texture, color, nutritional value, and safety. Deteriorative products can pose health risks if consumed beyond their expiration date or if stored improperly. To prevent the wastage of deteriorative products and ensure their safety and quality, it is important to store and handle them properly, such as by keeping them at the right temperature, using them before their expiration date, and following recommended storage and handling instructions. Proper inventory management and tracking can also help to minimize spoilage and waste. In some cases, technologies such as refrigeration, preservation techniques, or modified atmospheric packaging can help extend the shelf life of deteriorating products. However, it is important to balance the benefits of these technologies against their environmental and economic costs and consider alternatives such as reducing food waste through better distribution, consumption patterns, and waste reduction strategies. Sarkar (2012) investigated an EOQ model that took into account the fact that a perishable product's deterioration rate rises during the time being passed and becomes 100% on the date of expiration. In order to create a model for economic order quantity based on an upstream pay in later facility to increase demands, Chen and Teng (2014) employed the rate of deterioration connected to maximum lifetime. Later, Wu et al. (2014) developed a supply chain model in accordance with the upstream pay-in-later facility offered by the supplier to the retailer, while the merchant provides his clients with a down- stream pay-in-later facility. In addition, Teng et al. (2016) investigated an advance-cash payment related inventory model for perishable items. Wu et al. (2018) broadened the scenario where the supplier approaches the retailer for an advance-cash-credit (ACC) payment and the merchant offers customers a credit payment. Li et al. (2017) deepened their investigation of the issue by incorporating a strategy on pricing and employing an analysis of cash flow for a deteriorating item. Li and Teng (2018) investigated the pricing along with lot-sizing choices for decaying commodities under conditions where the selling price influences the demand rate, the product's freshness, the reference price, and the displayed stocks. Shaikh et al. (2019a) explored the role of a discount frame on the storage decision of a perishable item, where the item deteriorates constantly over the storage period. Alshanbari et al. (2021) adopted an increasing decay rate with respect to the items' storage time for determining the inventory decisions for a perishable item. De and Bhattacharya (2022) formulated a production inventory model with deterioration under a fuzzy system. Rahman et al. (2022) studied the interval form of an all units quantity discount and then explored its consequences on inventory planning for a deteriorating item under interval uncertainty. Manna et al. (2022) further developed inventory models for deteriorating items under interval uncertainty and solved them by adopting several meta-heuristic algorithms. Recently, Rahman et al. (2023) studied an EOQ model for deteriorating items with price-dependent demand under an interval environment.

Industries are significant sources of carbon emissions, accounting for a large portion of global greenhouse gas emissions. These emissions come from various industrial processes, such as manufacturing, construction, and transportation. Fossil fuel combustion for energy production is a significant source of carbon emissions in the industrial sector. Industries also release carbon emissions through chemical processes, such as the production of cement, which involves high-temperature processes that release large amounts of carbon dioxide. The industrial sector is responsible for around 30% of global carbon emissions, making it a crucial area for reducing carbon emissions to mitigate climate change (The World Health Organization, 2016). To address this, industry can adopt cleaner and more efficient production processes, use renewable energy sources, implement carbon capture and storage technology, and adopt circular economy principles to reduce waste and emissions. Governments and international organizations also play a critical role in reducing industrial emissions through policies, regulations, and incentives to promote sustainable industrial practices and move towards an economy with low carbon emissions. According to the analysis of the impact of various emissions restrictions (Benjaafar et al., 2012), businesses could successfully cut their carbon emissions by implementing operational changes and working with other supply chain participants. In order to lower carbon emissions produced by businesses, He et al. (2015) calculated the ideal size of lot and emissions with carbon regulations: cap-and-trade (where governments charge businesses a specific amount for each tonne of emissions they create) and carbon tax. These allowances might be auctioned off to the highest bidder and then traded on secondary markets to determine a price for carbon. Because the demand rate depends on credit duration, Dye and Yang (2015) assessed the effects of environmental legislation and credit periods on inventory management. Xu et al. (2016) studied the collaborative price and production choices for several products under cap-and-trade and carbon tax legislation. By taking into account default risks from issuing pay in later facilities for newsvendor models, Tsao et al. (2017) expanded the model of Dye and Yang (2015). The usage of credit payments in a supply chain model is investigated by Aljazzar et al. (2018), who discovered that doing so will lower the supply chain's carbon emissions. Bai et al. (2019) investigated supply chain coordination and carbon emission reductions. Afterward, Pan et al. (2020) solved a technical collaboration issue with investments to reduce carbon emissions through sustainable production inventory. Ji et al. (2020) studied an optimization model for carbon emission reduction investment. Sundar et al. (2021) investigated the effects of mitigation options on the control of methane emissions caused by rice paddies and livestock populations to reduce global warming. Ruidas et al. (2021) developed a production inventory model with price-dependent demand and interval-valued carbon emission parameters. Yadav et al. (2022) designed a sustainable production system under the pollution reduction effort during the production process. Recently, Khan et al. (2023b), Manna et al. (2023), and Ruidas et al. (2023) have developed many kinds of inventory models considering carbon emission investment. The related literature is described in the following tabular form (cf. Table 1).

Based on key features, Table 1 compares the pertinent, already-existing models in the literature. To handle the uncertain behaviour in demand as well as the system's parameters, very few existing works incorporated interval uncertainty during investigating problems. However, the impacts of emissions and advance payment mechanisms have not been investigated by incorporating the interval uncertainty. For thorough sustainability-focused supply chain management, an interval-valued inventory model that incorporates emissions impact and advance payment systems is essential. With the aid of this model, companies may not only optimise inventory levels while taking into account their carbon footprint but also account for the financial effects of advance payments and how they affect cash flow. There is an obvious research gap here that needs to be filled. Companies can meet sustainability goals, reduce emissions, and effectively manage their financial resources by studying a model like this, allowing them to make decisions that strike a balance between environmental responsibility and financial prudence. These are all essential elements of a modern, ethical business strategy, which are particularly focused on in the present study.

In this work, payment methods and inventory choices are included under the category of sustainability problems. The article's goal is to obtain insight into how options of payment under the widely used regulation of carbon tax affect inventory selections for perishable goods in light of laws for reducing carbon emissions. Because of the unexpected behaviour of clients, uncertainty is a natural phenomenon. An interval-valued inventory model is developed where all relevant inventory parameters are taken into consideration as interval-valued. Furthermore, the seller offers interval-valued discounts to customers in exchange for upfront payments. The objective function is changed into an interval-valued one and the accompanying differential equation into an interval differential equation as a result of choosing interval values for the inventory parameters. To solve the differential equation with interval values, the parametric approach of the interval presentation is introduced. Interval order relations and MATHEMATICA software are employed to find the best-found solutions to the interval-valued objective function of our suggested model. One numerical example is solved in order to validate the proposed system, and the best course of action is then determined via a post-optimality analysis.

1.1. Motivation of This Study

There are various reasons and advantages to developing an inventory model that takes into account the effects of carbon emissions and advance payment mechanisms. These are a few of the main arguments in favour of such a model:

- Many companies are concentrating on lowering their carbon footprint as concerns about climate change and environmental effects continue to grow. Businesses can make more sustainable decisions and aid in climate change mitigation by including carbon emissions in their inventory models. A greener supply chain and a reduction in carbon emissions from inventory management can benefit the environment.
- To reduce carbon emissions, governments and regulatory organisations are increasingly putting laws and regulations into place. Companies can proactively match their operations with evolving legislation, ensuring compliance and avoiding any fines or reputational problems by including carbon emissions in the inventory model.
- Energy use and transportation-related activities frequently correlate with carbon emissions. Businesses can find opportunities to cut expenses associated with transportation and energy use by optimising inventory decisions based on carbon emissions. Consolidating orders or changing transportation schedules, for instance, can result in more effective operations and cost savings.
- Including advance payments in the inventory model has a number of advantages. Businesses can take advantage of discounts or preferred terms that suppliers provide in exchange for early payment to lower the cost of carrying inventory and enhance cash flow. Companies may assess the trade-off between these advantages and potential hazards by incorporating advance payment terms into the model, ensuring that the best decisions are made about inventory management.

In general, firms can simultaneously address environmental sustainability, regulatory compliance, cost reduction, consumer preferences, and financial rewards by developing an inventory model that takes the impact of carbon emissions and advance payment into account. Companies may optimise their operations, improve their reputation, and contribute to a more sustainable future by coordinating inventory decisions with these criteria.

1.2. Novelty of This Study

In the present work, an EOQ model is developed and analysed for a deteriorating product whose demand rate is interval-valued and dependent on its selling price. Also, the deterioration rate of the perishable products is considered interval-valued. In this proposed work, carbon emissions due to placing an order, carbon emissions associated with handling each unit multiplied by the order quantity, and carbon emissions for preservation are taken as interval values. The supplier gives the buyer an interval-valued price discount rate to encourage their willingness to pay in advance. Since the parameters are chosen as interval-valued, the objective function is changed into an interval-valued one, and the corresponding differential equation is changed into an interval differential equation.

Table 1. A Brief Literature Review Related to The Proposed Work with Respect to Its Key Features

Author(s)	Demand	Advance Payment	Deterioration	Carbon Emissions	Governing Equation
Taleizadeh (2014)	Credit period dependent	Yes	Yes	No	Crisp
Teng et al. (2016)	Constant	Yes	Yes	No	Crisp
Li et al. (2017)	Price dependent	Yes	Yes	No	Crisp
Wu et al. (2018)	Constant	Yes	Yes	No	Crisp
Li and Teng (2018)	Price, stock, and time dependent	No	No	No	Crisp
Shaikh et al. (2018)	Price and advertisement frequency dependent	No	Yes	No	Fuzzy
Khan et al. (2019)	Price dependent	Yes	Yes	No	Crisp
Mondal et al. (2019)	Stock and selling price dependent	No	No	No	Interval
Shaikh et al. (2019b)	Price dependent	Yes	Yes	No	Interval
Khan et al. (2020)	Stock and price dependent	Yes	Yes	No	Crisp
Rahman et al. (2020)	Imprecise and price dependent	No	No	Yes	Interval
Ruidas et al. (2021)	Price dependent	No	No	No	Crisp
Manna et al. (2022)	Price dependent	Yes	Yes	No	Interval
Ruidas et al. (2022)	Price and green level dependent	No	No	Yes	Crisp
Rahman et al. (2022)	Stock and price dependent	No	Yes	No	Interval
Mondal et al. (2023)	Price dependent	Yes	Yes	No	Interval
Ruidas et al. (2023)	Price and green level dependent	No	No	Yes	Interval
Chaudhari et al. (2023)	Advertisement, price and stock dependent	No	Yes	Yes	Crisp
Yang (2023)	Time-varying	No	Yes	No	Crisp
Khan et al. (2023a)	Price and time dependent	Yes	No	No	Crisp
This work	Price dependent	Yes	Yes	Yes	Interval

The organization of the remaining part of the manuscript is given as follows: Notation and assumptions are presented in Section 2 to formulate the model. The mathematical formulation of the model for advance payment is delivered in Section 3. The solution procedure for the optimzation problem is provided in Section 4, and to validate our proposed model, one numerical example is considered in Section 5. To get some managerial insights, sensitivity analyses are executed in Section 6. To maximize profitability, several insights are provided in Section 7, while the major findings along with future research opportunities are delivered in Section 8.

2. Notation and Assumptions

Notation of this study is listed in Appendix A. The following fundamental assumptions are considered to construct the corresponding EOQ model:

(i) The interval-valued demand rate of the customers is taken as a decreasing function of the selling price of the product. Mathematically, the interval-valued demand rate is defined as: $[D_L(p), D_U(p)] = [\alpha_{1L}, \alpha_{1U}] - [\beta_{1L}, \beta_{1U}]p = [\alpha_{1L} - \beta_{1U}p, \alpha_{1U} - \beta_{1L}p]$, where $\alpha_{1L}, \beta_{1L} > 0$.

(ii) The interval-valued deterioration rate of perishable goods is given by: $[\theta_{1L}, \theta_{1U}]$, $0 \le \theta_{1L} < \theta_{1U} << 1$.

(iii) Perished products have no replacement, repair, or salvage value in this model.

(iv) The supplier makes a restriction to the buyer that they have to cover the full cost of the purchase up front in n evenly spaced installments within the lead L_1. In this instance, the supplier provides an interval-valued discount rate $[r_L, r_U]$ on the entire cost of the purchase.

(v) The total interval-valued fixed carbon emissions per cycle of replenishment includes interval-valued carbon emissions due to placing an order $[K_{1L}, K_{1U}]$, interval-valued amount of carbon emissions associated with purchasing each unit $[c_{1pL}, c_{1pU}]$ multiplied by the order quantity $[Q_L, Q_U]$, and interval-valued amount of carbon emissions for storing the interval-valued level of inventory $[x_{1L}(t), x_{1U}(t)]$.

(vi) Infinite time horizon is considered, and shortages are disallowed.

3. Mathematical Model

In this model, an order of $[Q_L, Q_U]$ units of products are arrived in stock at $t = 0$. After that, the stock level decreases due to the customers' demand and also deteriorates at an interval-valued deterioration rate $[\theta_{1L}, \theta_{1U}]$, and the inventory level becomes zero at the end of the cycle (see Figure 1). Apparently, the governing equation of the inventory level $[x_{1L}(t), x_{1U}(t)]$ is interval-valued, and at any time $t \in [0, T]$, it is given as follows:

$$\frac{d\left[x_{1L}(t), x_{1U}(t)\right]}{dt} + \left[\theta_{1L}, \theta_{1U}\right]\left[x_{1L}(t), x_{1U}(t)\right] = -\left[D_L(p), D_U(p)\right], \ t \in [0, T] \tag{1}$$

with the conditions $[x_{1L}(0), x_{1U}(0)] = [Q_L, Q_U]$ and $[x_{1L}(T), x_{1U}(T)] = [0, 0]$.

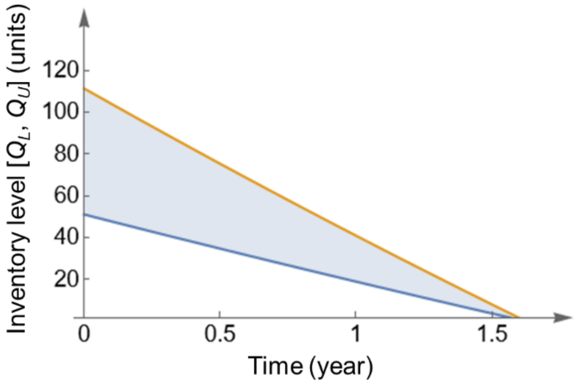

Figure 1. Pictorial representation of interval-valued inventory level.

Using the parametric approach, the Equation (1) can be converted into the following differential equation:

$$\frac{dx_1(\eta_1, t)}{dt} + \theta_1(\eta_2)x_1(\eta_1, t) = -D(\eta_3, p) \tag{2}$$

with the boundary conditions $x_1(\eta_1, 0) = Q(\eta_4)$ and $x_1(\eta_1, T) = 0$, where $x_1(\eta_1, t) = x_{1L}(t) + \eta_1\{x_{1U}(t) - x_{1L}(t)\}$, $\theta_1(\eta_2) = \theta_{1L} + \eta_2(\theta_{1U} - \theta_{1L})$, $D(\eta_3, p) = D_L(p) + \eta_3\{D_U(p) - D_L(p)\}$, and $Q(\eta_4) = Q_L + \eta_4(Q_L - Q_L)$; $\eta_1, \eta_2, \eta_3, \eta_4 \in [0, 1]$.

Using the boundary condition $x_1(\eta_1, T) = 0$, the solution of the differential Equation (2) is given by:

$$x_1(\eta_1, t) = \frac{D(\eta_3, p)}{\theta_1(\eta_2)}\left\{e^{\theta_1(\eta_2)(T-t)} - 1\right\} \tag{3}$$

Therefore, converting the parametric form into an interval, the lower and upper bounds of the inventory level during the interval of time $[0, T]$ is calculated as:

$$x_{1L}(t) = \frac{D_L(p)}{\theta_{1U}}\left\{e^{\theta_{1L}(T-t)} - 1\right\} \tag{4}$$

$$x_{1U}(t) = \frac{D_U(p)}{\theta_{1L}}\left\{e^{\theta_{1U}(T-t)} - 1\right\} \tag{5}$$

Now, from Equation (3), using the condition $x_1(\eta_1, 0) = Q(\eta_4)$, the initial stock level is given by:

$$Q(\eta_4) = \frac{D(\eta_3, p)}{\theta_1(\eta_2)}\left\{e^{\theta_1(\eta_2)T} - 1\right\} \tag{6}$$

Therefore, the interval-valued stock level is expressed as follows:

$$[Q_L, Q_U] = \left[\frac{D_L(p)}{\theta_{1U}}\left\{e^{\theta_{1L}T} - 1\right\}, \frac{D_U(p)}{\theta_{1L}}\left\{e^{\theta_{1U}T} - 1\right\}\right] \tag{7}$$

Hence $Q_L = \frac{D_L(p)}{\theta_{1U}}\left\{e^{\theta_{1L}T} - 1\right\}$ and $Q_U = \frac{D_U(p)}{\theta_{1L}}\left\{e^{\theta_{1U}T} - 1\right\}$.

Sales revenue is given by:

$$\left[SR_L(T, p), SR_U(T, p)\right] = p\int_0^T \left[D_L(p), D_U(p)\right]dt = \left[p(\alpha_{1L} - \beta_{1U}p)T, p(\alpha_{1U} - \beta_{1L}p)T\right] \tag{8}$$

Total interval-valued cost for ordering per replenishment cycle is:

$$\left[OC_L, OC_U\right] = \left[O_L, O_U\right] \tag{9}$$

Holding cost per replenishment cycle is presented by:

$$\left[HC_L(T, p), HC_U(T, p)\right] = \begin{bmatrix} h_{cL}\dfrac{D_L}{\theta_{1U}}\left\{\dfrac{1}{\theta_{1U}}\left(e^{\theta_{1L}T} - 1\right) - T\right\}, \\ h_{cU}\dfrac{D_U}{\theta_{1L}}\left\{\dfrac{1}{\theta_{1L}}\left(e^{\theta_{1U}T} - 1\right) - T\right\} \end{bmatrix} \tag{10}$$

The total interval-valued portion of carbon emission during per cycle of replenishment is presented by:

$$\left[CE_L(T, p), CE_U(T, p)\right] = \left[K_{1L}, K_{1U}\right] + \left[c_{1pL}, c_{1pU}\right]\left[Q_L, Q_U\right] + \left[c_{1hL}, c_{1hU}\right]\int_0^T \left[x_L(t), x_U(t)\right]dt = \begin{bmatrix} K_{1L} + c_{1pL}Q_L + c_{1hL}\dfrac{D_L}{\theta_{1U}}\left\{\dfrac{1}{\theta_{1U}}\left(e^{\theta_{1L}T} - 1\right) - T\right\}, \\ K_{1U} + c_{1pU}Q_U + c_{1hU}\dfrac{D_U}{\theta_{1L}}\left\{\dfrac{1}{\theta_{1L}}\left(e^{\theta_{1U}T} - 1\right) - T\right\} \end{bmatrix} \tag{11}$$

Now, the seller asks the buyer to prepay the purchasing cost $[PC_L(p, T), PC_U(p, T)]$ with n_1 equal installments during L_1 before receiving the items in the case of advance payment. The supplier typically gives the buyer a price discount of $[r_L, r_U]$ to encourage their willingness to pay in advance.

Consequently, the buyer's purchasing cost per replenishment cycle is calculated as:

$$\left[PC_L(T,p), PC_U(T,p)\right] = \left(1 - \left[r_L, r_U\right]\right)\left[c_{pL}, c_{pU}\right]\left[Q_L, Q_U\right] = \begin{bmatrix} (1 - r_U)c_{pL}\dfrac{D_L(p)}{\theta_{1U}}\left\{e^{T\theta_{1L}} - 1\right\}, \\ (1 - r_L)c_{pU}\dfrac{D_U(p)}{\theta_{1L}}\left\{e^{T\theta_{1U}} - 1\right\} \end{bmatrix} \tag{12}$$

The interval-valued capital cost per replenishment cycle prior to the delivery for the advance payment is as follows:

$$\left[CC_L(T,\,p),\;CC_U(T,\,p)\right]=$$
$$\left[I_{cL},\,I_{cU}\right]\frac{\left[PC_L,\,PC_U\right]}{n_1}\frac{L_1}{n_1}\left(1+2+\ldots+n_1\right)=$$
$$\left[\frac{(1+n_1)L_1}{2n_1}I_{cL}PC_L,\;\frac{(1+n_1)L_1}{2n_1}I_{cU}PC_U\right] \tag{13}$$

Furthermore, the interest charged for on-hand inventory in interval form is given by:

$$\left[IC_L(T,p),\;IC_U(T,p)\right]=$$
$$\left[I_{cL},\,I_{cU}\right]\left(1-\left[r_L,\,r_U\right]\right)\left[c_{pL},\,c_{pU}\right]\int_0^T\left[x_{1L},\,x_{1U}\right]dt=$$
$$\begin{bmatrix}\left(1-r_U\right)I_{cL}c_{pL}\dfrac{D_L}{\theta_{1U}}\left\{\dfrac{1}{\theta_{1U}}\left(e^{\theta_{1L}T}-1\right)-T\right\}, \\ \left(1-r_L\right)I_{cU}c_{pU}\dfrac{D_U}{\theta_{1L}}\left\{\dfrac{1}{\theta_{1L}}\left(e^{\theta_{1U}T}-1\right)-T\right\}\end{bmatrix} \tag{14}$$

Combining the results from Equations (8) ~ (14), the total average profit function per cycle of replenishment is given by:

$$\left[TP_L(T,\,p),\;TP_U(T,\,p)\right]=$$
$$\begin{bmatrix}SR_L-O_U-HC_U-\eta_U CE_U-PC_U-IC_U-CC_U, \\ SR_U-O_L-HC_L-\eta_L CE_L-PC_L-IC_L-CC_L\end{bmatrix}=$$
$$\begin{bmatrix}p\left(\alpha_{1L}-\beta_{1U}p\right)T-O_U-h_{cU}\dfrac{D_U}{\theta_{1L}}\left\{\dfrac{1}{\theta_{1L}}\left(e^{\theta_{1U}T}-1\right)-T\right\} \\ -\eta_U\left(K_{1U}+c_{1pU}Q_U+c_{1hU}\dfrac{D_U}{\theta_{1L}}\left\{\dfrac{1}{\theta_{1L}}\left(e^{\theta_{1U}T}-1\right)-T\right\}\right) \\ -\left(1-r_L\right)c_{pU}\dfrac{D_U(p)}{\theta_{1L}}\left\{e^{\theta_{1U}T}-1\right\}-\dfrac{(1+n_1)L_1}{2n_1}I_{cU}PC_U \\ -\left(1-r_L\right)I_{cU}c_{pU}\dfrac{D_U}{\theta_{1L}}\left\{\dfrac{1}{\theta_{1L}}\left(e^{\theta_{1U}T}-1\right)-T\right\}, \\ p\left(\alpha_{1U}-\beta_{1L}p\right)T-O_L-h_{cL}\dfrac{D_L}{\theta_{1U}}\left\{\dfrac{1}{\theta_{1U}}\left(e^{\theta_{1L}T}-1\right)-T\right\} \\ -\eta_L\left(K_{1L}+c_{1pL}Q_L+c_{1hL}\dfrac{D_L}{\theta_{1U}}\left\{\dfrac{1}{\theta_{1U}}\left(e^{\theta_{1L}T}-1\right)-T\right\}\right) \\ -\left(1-r_U\right)c_{pL}\dfrac{D_L(p)}{\theta_{1U}}\left\{e^{\theta_{1L}T}-1\right\}-\dfrac{(1+n_1)L_1}{2n_1}I_{cL}PC_L \\ -\left(1-r_U\right)I_{cL}c_{pL}\dfrac{D_L}{\theta_{1U}}\left\{\dfrac{1}{\theta_{1U}}\left(e^{\theta_{1L}T}-1\right)-T\right\}\end{bmatrix} \tag{15}$$

Therefore, the interval-valued average profit of the system per replenishment cycle is presented by:

$$\left[AP_L(T,p),AP_U(T,p)\right]=\left[\frac{TP_L(T,p)}{T},\;\frac{TP_U(T,p)}{T}\right] \tag{16}$$

As a result, the centre and radius of the average profit are given by:

$$AP_c(T,p)=\frac{AP_U(T,p)+AP_L(T,p)}{2} \tag{17}$$

$$AP_r(T,p)=\frac{AP_U(T,p)-AP_L(T,p)}{2} \tag{18}$$

Then, the corresponding optimization problem is given by:

$$Maximize\ AP_c(T,p),\;T\ge 0\ and\ p\ge 0 \tag{19}$$

4. Solution Procedure

Rahman et al. (2020) proposed the *c-r* optimization (centre-radius optimization) technique, which is based on the interval order relation suggested by Bhunia and Samanta (2014). In this method, an interval-valued optimization problem is converted into a crisp one. It is the method in which either the center of the objective function is optimized or the radius of the objective function is optimized (if the center of the objective function is constant valued). The definitions of maximizer (local and global) are defined with respect to Bhunia and Samanta (2014) interval order relation to derive this technique.

Definition 1. Let use consider two intervals of real numbers $A_1=[a_{1L},a_{1U}]\cong\langle a_{1c},\,\mathrm{a}_{1r}\rangle$ and $B_1=[b_{1L},b_{1U}]\cong\langle b_{1c},\,b_{1r}\rangle$. Then, the interval order relations between A_1 and B_1 are defined as: $A_1\ge max\ B_1\Leftrightarrow\begin{cases}a_{1c}>b_{1c},\text{ if }a_{1c}\ne b_{1b}\\ a_{1r}\le b_{1r},\text{ if }a_{1c}=b_{1b}\end{cases}$ and $A_1>max\ B_1$ $\Leftrightarrow A_1\ge max\ B_1$ and $A_1\ne B_1$.

Definition 2. A point $(T^*,p^*)\in E_1\times E_2$ is called a local maximizer of $[AP_L(T,P),AP_U(T,p)]$ if there exists a positive δ_1 that: $[AP_L(T^*,p^*),AP_U(T^*,p^*)]\ge min\,[AP_L(T,p),AP_U(T,p)]$, $\forall\ (T,p)\in(E_1\times E_2)\cap B_1\left(\left(T^*,p^*\right),\;\delta_1\right)$, where $B_1((T^*,p^*),\delta_1)$ is the open ball with centre at (T^*,p^*) and radius δ_1.

Definition 3. A point $(T^*,p^*)\in E_1\times E_2$ is called a global maximizer of $[AP_L(T,p),AP_U(T,p)]$ if $[AP_L(T^*,p^*),AP_U(T^*,p^*)]\ge min\,[AP_L(T,p),AP_U(T,p)]$, $\forall(T,p)\in E_1\times E_2$.

4.1. Centre-Radius Optimization Technique

Theorem 1: The interval-valued function: $[AP_L(T,p),AP_U(T,p)]\equiv\langle AP_c(T,p),AP_r(T,p)\rangle$ has a maximizer at $T=T^*$ and $p=p^*$ if and only if $AP_c(T,P)$ has a maximizer at (T^*,p^*), if $AP_c(T,P)\ne$ constant; $AP_r(T,P)$ has a minimizer at (T^*,p^*), if $AP_c(T,P)=$ constant.

Proof: $T=T^*$, $p=p^*$ is said to be the global maximal point of $AP_c(T,p)$ if and only if:

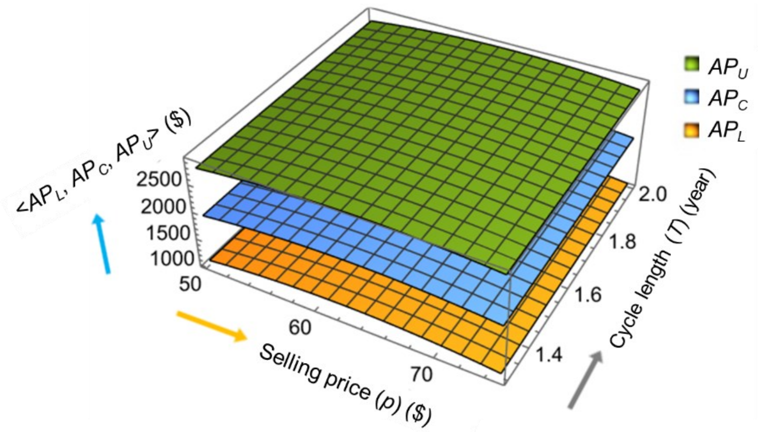

Figure 2. Interval-valued average profit of Example 1.

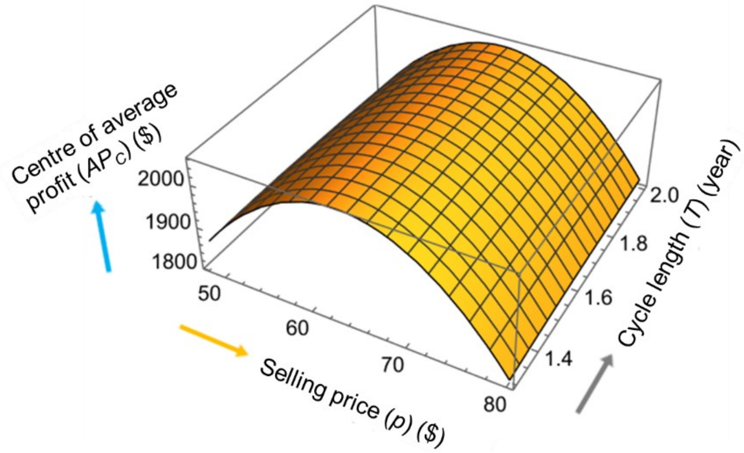

Figure 3. Concavity of the centre of average profit (AP_c) with respect to the selling price (p) and the length of the replenishment cycle (T) of Example 1.

$$AP_c\left(T^*,p^*\right) \geq max\ AP_c\left(T,p\right) \Leftrightarrow$$

$$\begin{cases} AP_c\left(T^*,p^*\right) > AP_c\left(T,p\right), \\ \text{if } AP_c\left(T^*,p^*\right) \neq AP_c\left(T,p\right), \forall T,p>0 \\ AP_r\left(T^*,p^*\right) \leq AP_r\left(T,p\right), \\ \text{if } AP_c\left(T^*,p^*\right) = AP_c\left(T,p\right), \forall T,p>0 \end{cases} \Leftrightarrow$$

$$\begin{cases} AP_c\left(T^*,p^*\right) > AP_c\left(T,p\right), \text{ if } AP_c\left(T,p\right) \neq \text{constant} \\ AP_r\left(T^*,p^*\right) \leq AP_r\left(T,p\right), \text{ if } AP_c\left(T,p\right) = \text{constant} \end{cases} \Leftrightarrow$$

$$\begin{cases} AP_c\left(T,p\right) \text{ has a maximizer at } (T^*,p^*), \\ \text{if } AP_c\left(T,p\right) \neq \text{constant} \\ AP_r\left(T,p\right) \text{ has a minimizer at } (T^*,p^*), \\ \text{if } AP_c\left(T,p\right) = \text{constant} \end{cases} \tag{20}$$

This concludes the proof.

5. Numerical Illustration

Since the objective function, the centre of average profit (AP_c), is highly nonlinear in nature with respect to the selling price (p) and length of the replenishment cycle (T), the optimization problem (19) cannot be solved analytically. To check the reality and validate the proposed model in the interval environment, a numerical example is considered, and then MATHEMATICA software is used to find the best-found solution to that example. In addition, we have drawn some concavity graphs to show the concavity of the centre of the average profit function with respect to different decision variables in accordance with the considered example.

Example 1: The following values of the different inventory parameters related to the proposed model are considered: $[\alpha_{1L}, \alpha_{1U}] = [95, 100]$ (in unit); $[\beta_{1L}, \beta_{1U}] = [0.8, 0.9]$; $[\theta_{1L}, \theta_{1U}] = [0.07, 0.09]$; $[c_{pL}, c_{pU}] = [10, 12]$ (in \$); $[O_L, O_U] = [170, 175]$ (in \$); $[h_{cL}, h_{cU}] = [0.8, 1]$ (in \$); $[K_{1L}, K_{1U}] = [180, 190]$ (in unit); $[c_{1pL}, c_{1pU}] = [4, 6]$ (in unit); $[c_{1hL}, c_{1hU}] = [2, 3]$ (in unit); $[I_{cL}, I_{cU}] = [0.09, 0.11]$; $[\eta_L, \eta_U] = [0.09, 0.10]$; $[r_L, r_U] = [0.4, 0.5]$; $L_1 = 0.5$ year; $n_1 = 5$.

Table 2. Optimal Solutions of Example 1

Variables/Unknown Parameters	Optimal Values
p	\$63.133
T	1.626 years
$<AP_c, AP_r>$	<\$2057.887, \$874.329>
$[AP_L, AP_U]$	[\$1183.557, \$2932.216]
$[Q_L, Q_U]$	[51.145, 111.433]

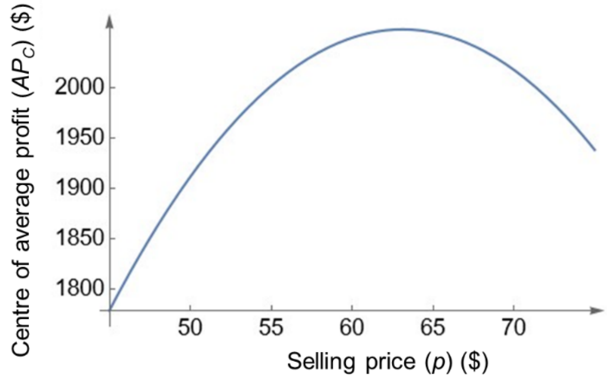

Figure 4. Concavity of the centre of average profit (AP_c) with respect to the selling price (p) of Example 1.

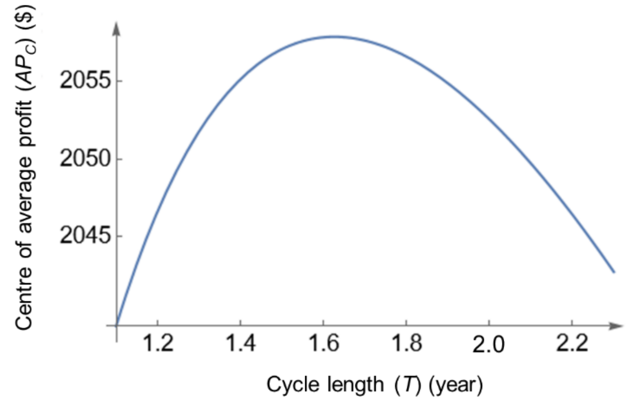

Figure 5. Concavity of the centre of average profit (AP_c) with respect to the length of the replenishment cycle (T) of Example 1.

The best-found solutions for different decision variables and unknown parameters in Example 1 are presented in the following Table 2. Furthermore, the nature of the objective function, the centre of average profit (AP_c), is explored graphically in Figures 2 ~ 5 with respect to different decision variables in accordance with the considered Example 1.

Now, the concavity of the centre of the average profit function $AP_c(p, T)$ is shown numerically by the eigenvalues of its Hessian matrix **H**. The Hessian matrix **H** for the centre of the average profit $AP_c(p, T)$ for Example 1 w. r. to optimal values of selling price (p) and cycle length (T), i.e., at (p^*, T^*) = (63.132652, 1.626119) is given by:

$$\mathbf{H}=\begin{pmatrix}\dfrac{\partial^2 AP_c}{\partial p^2} & \dfrac{\partial^2 AP_c}{\partial p \partial T}\\ \dfrac{\partial^2 AP_c}{\partial T \partial p} & \dfrac{\partial^2 AP_c}{\partial T^2}\end{pmatrix}_{(p,\,T)=\left(p^*,\,T^*\right)} =\begin{pmatrix}-1.700000 & 1.250878\\ 1.250878 & -92.623168\end{pmatrix} \tag{21}$$

Consequently, the eigenvalues of the Hessian matrix **H** are given by $\lambda_1^* = -92.640374$ and $\lambda_2^* = -1.682794$, which are all negative. Therefore, the Hessian matrix **H** of the centre of the average profit $AP_c(p, T)$ w. r. to optimal values of the decision variables is strictly negative definite. Hence, $AP_c(p, T)$ is strictly concave w. r. to p, T, and therefore, $AP_c(p, T)$ is maximum at (p^*, T^*) = (63.132652, 1.626119).

6. Sensitivity Analysis

To study the effect of changes in the different parameters of the proposed model on the centre of average profit (AP_c), selling price (p), and replenishment cycle (T), the sensitivity analyses are executed in Example 1. In Figures 6(a) ~ 6(f), the best outcomes of these studies are represented graphically. From the above sensitivity figures, the following implications are observed: (i) From Figure 6(a), it is seen that AP_c is highly sensitive, whereas p and T are sensitive moderately against the changes of the interval-valued fixed demand $[a_{1L}, a_{1U}]$. Here, T is reversely affected by the changes in $[a_{1L}, a_{1U}]$. (ii) Again, Figure 6(b) represents that AP_c is highly sensitive in a negative way and p is moderately sensible based on the change in the interval-valued parameter of demand $[\beta_{1L}, \beta_{1U}]$. In addition, T is insensitive with respect to the demand parameter $[\beta_{1L}, \beta_{1U}]$. (iii) Here, we discuss the effect of changes in the interval-valued purchasing cost $[c_{pL}, c_{pU}]$ of the product on AP_c, p, and T. From Figure 6(c), it is observed that AP_c, p, and T are insensitive with respect to the interval-valued purchasing cost $[c_{pL}, c_{pU}]$. (iv) Furthermore, from Figures 6(d) and 6(e), it is seen that AP_c, p, and T are insensitive with respect to the changes in the interval-valued holding cost $[h_{cL}, h_{cU}]$ and the interval-valued carbon emitted tax $[\eta_L, \eta_U]$. (v) Finally, we discuss the effect of the changes in the interval valued ordering cost $[O_L, O_U]$. Figure 6(f) shows that T is less sensitive, whereas AP_c and p are insensitive with respect to $[O_L, O_U]$.

7. Managerial Insights

Reducing carbon emissions is becoming increasingly important for organizations as they work to meet sustainability goals and reduce their impact on the environment. In addition, the selling price is a critical factor for any business, as it directly influences the company's profitability and competitiveness in the market. Here are some managerial insights about carbon emissions reduction and selling price:

(i) When evaluating carbon emission reduction technologies, it's important to consider the entire lifecycle impact of the technology, including its production, use, and disposal. For ex-

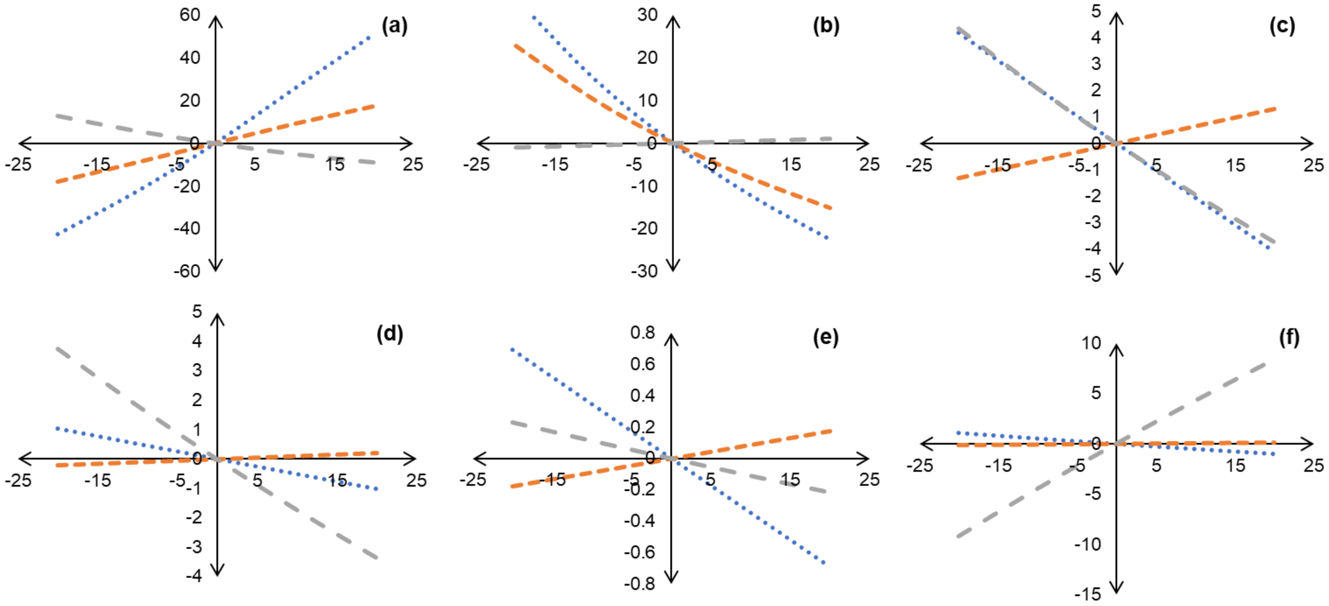

Figure 6. The effect of changes of different parameters (a) [α_{1L}, α_{1U}], (b) [β_{1L}, β_{1U}], (c) [h_{cL}, h_{cU}], (d) [c_{pL}, c_{pU}], (e) [η_L, η_U], and (f) [O_L, O_U] on AP_c, p, and T.

ample, while electric vehicles produce fewer emissions while in use, the production of batteries and charging infrastructure can have a significant carbon footprint.

(ii) Transitioning to renewable energy sources such as wind, solar, or hydropower can significantly reduce a company's carbon emissions. Investing in renewable energy sources can also provide long-term cost savings as the cost of renewable energy continues to decrease.

(iii) Companies can reduce their carbon emissions by adopting energy-efficient practices such as using LED lighting, installing energy-efficient HVAC systems, and improving insulation. These practices can help to reduce energy consumption and save money on energy bills.

(iv) Employee engagement is critical to the success of any sustainability initiative. Encouraging employees to adopt energy-efficient practices, such as turning off lights and computers when not in use, can help to reduce a company's carbon footprint.

(v) Companies can work with their suppliers to reduce their carbon footprint by encouraging them to adopt sustainable practices, such as using renewable energy and reducing waste. Collaboration with suppliers can help to create a more sustainable supply chain and reduce the overall carbon emissions associated with a company's products or services.

(vi) Research the market and analyze the prices of similar products or services. Understanding the competition and consumer demand will help determine the optimal selling price. However, make sure to differentiate your product or service from competitors to avoid being seen as a low-price option.

(vii) The market is dynamic, and the price that was profitable yesterday may not be sustainable tomorrow. Stay flexible and adapt your selling price accordingly to ensure your business remains competitive.

(viii) The selling price should reflect the value proposition of the product or service. Communicate the unique features, benefits, and value proposition to the customer to justify the selling price.

(ix) Monitor the selling price and adjust it as needed to align with market trends, changes in the cost structure, and customer feedback.

Finally, reducing carbon emissions requires a comprehensive approach that involves evaluating technology options, adopting energy-efficient practices, and collaborating with the stakeholders. By taking a holistic approach to sustainability, companies can make a significant impact on reducing their carbon emissions and contribute to a more sustainable future. In addition, setting the right selling price requires a thorough understanding of costs, market conditions, and customer value. By applying these managerial insights, businesses can maximize profitability while remaining competitive in the market.

8. Conclusions and Future Scope

In this study, an interval-valued inventory model is developed where all relevant inventory factors are taken into account. Moreover, the seller offers interval-valued discounts to customers in exchange for upfront payments. The objective function is changed into an interval-valued objective and the accompanying differential equation into an interval differential equation as a result of choosing interval values for the inventory parameters. An interval differential equation with a parametric approach is introduced, and the interval-valued objective function is constructed to solve the differential equation with interval values. With the aid of MATHEMATICA software and interval order relations, the goal function with interval values is solved. One numerical example is solved, and a sensitivity analysis of the best course of action is carried out in order to validate the proposed model.

As worries about climate change and its effects on the environment continue to rise, more businesses are focusing on reducing their carbon footprint. By including carbon emissions in the inventory model, businesses can make more environmentally friendly decisions and contribute to mitigating climate change. The environment can benefit from a greener supply chain and a decrease in carbon emissions through inventory management. This study provides proper inventory decisions to make supply coordination much greener by curbing emissions from several operations.

There are several benefits to including advance payment in the inventory model. To reduce the cost of carrying inventory and improve cash flow, businesses might take advantage of discounts or preferred terms that suppliers provide in the exchange for early payment. By including advance payment terms into the model, businesses may evaluate the trade-off between these benefits and potential risks, ensuring that the best decisions are made about inventory management. Anybody can add nonlinear stock-dependent demand, use preservation technology, employ carbon cap and trade policy, trade credit (both single and two-level), etc. for further exploration. Also, anyone can use the soft computing technique to resolve this kind of challenging issue.

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Appendix A. Notation

Notation	Description
$[x_{1L}(t), x_{1U}(t)]$	Interval-valued inventory status at time t (units)
$[\alpha_{1L}, \alpha_{1U}]$	Interval-valued fixed demand (units)
$[\beta_{1L}, \beta_{1U}]$	Interval-valued demand parameter
$[Q_L, Q_U]$	Interval-valued order quantity of the buyer (units)
$[\theta_{1L}, \theta_{1U}]$	Interval-valued deterioration rate, $0 \le \theta_{1L} < \theta_{1U} << 1$
$[h_{cL}, h_{cU}]$	Interval-valued holding cost (\$/unit)
$[c_{1hL}, c_{1hU}]$	Interval-valued portion of carbon emissions in inventory
$[c_{pL}, c_{pU}]$	Interval-valued purchasing cost of an item (\$/unit)
$[c_{1pL}, c_{1pU}]$	Interval-valued amount of carbon emissions related with per unit purchased
$[CE_L, CE_U]$	Interval-valued total portion of carbon emission per cycle of replenishment
$[IC_L, IC_U]$	Interval-valued interest charged (\$/time)
$[O_L, O_U]$	Interval-valued ordering cost (\$/order)
$[K_{1L}, K_{1U}]$	Interval-valued portion of carbon emission per order
L_1	Duration of advance payment in units of time
n_1	Equal installments number in the advance payment mechanism
$[r_L, r_U]$	Interval-valued discount rate
$[\eta_L, \eta_U]$	Interval-valued carbon emitted tax paid per unit
$[TP_L, TP_U]$	Interval-valued total profit function of the system (\$)
$[AP_L, AP_U]$	Interval-valued total average profit of the system (\$/year)
$<AP_c, AP_r>$	*c-r* form of average profit function of the system
Decision Variables	
p	The selling price of each product (\$/unit)
T	Length of the buyer's replenishment cycle in units of time (year)

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